Financial Innovation, Collateral Hedging and Macro-prudential Policies

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Abstract

This paper features an economy with incomplete markets, collateralised lending and investors that hold heterogeneous beliefs about the states of the world. Financial innovation takes the form of collateral hedging, which enables collateral protection insurance (CPI) contracts together with existing investment in capital to back loans. The contribution of this paper is to characterize the effectiveness of two macro-prudential policies, namely higher collateral requirements on CPI or capital, towards mitigating default friction and maximizing social welfare. The degree of belief disagreement is an important factor determining the effectiveness of policy. When disagreement is extreme, then the policy via financial innovation is not effective, unless it is not costly, and macro-prudential interventions call for higher collateral requirements on capital; while under moderate levels of disagreement, increasing collateral requirements on capital is not effective, and policy interventions call for financial innovation.

1 Introduction

In recent years, the corporate bond market has experienced clustered default events with adverse effects on macro-economic outcomes (Arellano, 2008; Giesecke, Longstaff, Schaefer, & Strebulaev, 2011). Major central banks have resorted to macro-prudential regulation in order to limit the exposure of corporations to aggregate risk. One important macro-prudential policy tool is to set a minimum requirement on haircuts (Basel, 2010; Longworth, 2010). The haircut term refers to the percentage difference between the market value of an asset and the amount held as loan collateral.[1] However, high requirements on haircuts weaken the pledge-ability of collateral and reduce private investment (Mendicino et al., 2012). This effect can be reflected in the I/S ratio, namely the ratio of private nonresidential fixed investment to the gross private saving. Low I/S ratio means that a small proportion of the private sector saving flows to the capital investment, such as commercial real estate, tools, machinery, and factories. The blue line in the figure 1 shows that the financial crisis in the US drives the investment-saving ratio down, but after 2008, the government and policy interventions did not increase the I/S ratio. What blocked the conduit between private saving and capital investment during and especially after the crisis? Are there

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[1]Data on haircuts are not available, since some corporations, especially small and medium sized enterprises, do not publish information about their external funding.

any other appropriate macro-prudential policies to stimulate the economy other than imposing higher requirements on haircuts?

To address these question, this paper looks into the collateralized lending by evaluating the manner of asset-backed commercial paper (ABCP) (Gertler & Gilchrist, 2018). ABCP which funds mortgages, auto loans and credit card debt facilitates the flow of funds between corporations and investors (Bens & Monahan, 2008). The green line in Figure 1 shows that the ABCP spread went up from 2007: I, reflecting that the short-term lending market contracted before the bankruptcy of Lehman Brothers in 2008: IV. This significant increase in 2007 and 2008 could be attributed to the lenders' perceived reduction of collateral value. In 2009, the drop of ABCP spread reflected the similarity of beliefs among borrowers and lenders about the value of borrowers' collateral. Thus, the reason for the decline in the I/S ratio during the recession could be the fear of lenders about the plunge of the collateral value. However, the slow recovery afterwards could be attributed to limited pledge-ability of collateral, notwithstanding macro-prudential policies implemented during that period. The preceding argument motivates this paper to study how the degree of belief heterogeneity among investors impacts the effectiveness of macro-prudential policies. The contribution of this paper is to characterize two types of macro-prudential policies: (i) perturbations on haircuts through regulations of collateral requirements, (ii) financial innovation based on collateral protection insurance (CPI). I show that increasing haircuts does not always lead to higher social welfare, measured by the sum of heterogeneous agents' expected utilities. Specifically, if increasing haircuts lowers aggregate risk in the economy, eliminating risk-sharing opportunities, then social welfare reduces.

In the present setting, I take my cue from the work of Geanakoplos (2003), who models collateral and default in the form of repayment enforce-ability problems. I consider a two-period economy with uncertainty represented by two states of the world in the second period, boom and bust, and three types of agents that hold heterogeneous beliefs about the realization of the bust state: optimists, moderates and pessimists. Agents can borrow from each other by trading financial contracts. The definition of financial contracts is borrowed from Geanakoplos and Zame (2014), who introduce repayment enforce-ability problems and collateralized lending. If the contract sellers default, then they deliver the collateral to the buyers. Optimists have access to a technology that allows them to invest in capital and enhance their state-contingent wealth; while moderates and pessimists are constrained by technology barriers and do not have access to this technology. Thus, optimists can borrow funds from both moderates and pessimists in order to invest, by pledging a fraction of their investment as collateral. This fraction is controlled by the macro-prudential regulatory framework of the economy. Macro-prudential interventions affect indirectly the trade of assets in the economy and induce an alternative allocation of resources that cannot be attained by the market.

I also consider financial innovation as an alternative macro-prudential policy tool. Specifically, financial innovation is based on collateral protection insurance (CPI) that has been popularized in the automobile industry. My hypothesis is that this is a relevant instrument because company liquidations can be thought of as faulty vehicles that have lost market value. When households borrow with the aim to buy a car, lenders require them to buy also vehicle insurance like CPI. Without this requirement, once the car is damaged, the borrowers have an incentive to default and the lenders end up with a car that is worthless. When the CPI is exercised, the sellers of those contracts transfer resources to borrowers, so that they can repay their debts. As a result, CPI trading influences the vehicle loan market. Figure 2 provides evidence on CPI trades and shows that the fluctuation of vehicle loan volumes coincide with the fluctuation in the amount of vehicle insurance in the US. Thus, if my hypothesis is correct, macro-prudential policies inducing optimists to buy CPI, can influence the trading of corporate bonds. In the present setting, CPI trading requires optimists to buy additional collateral protection insurance

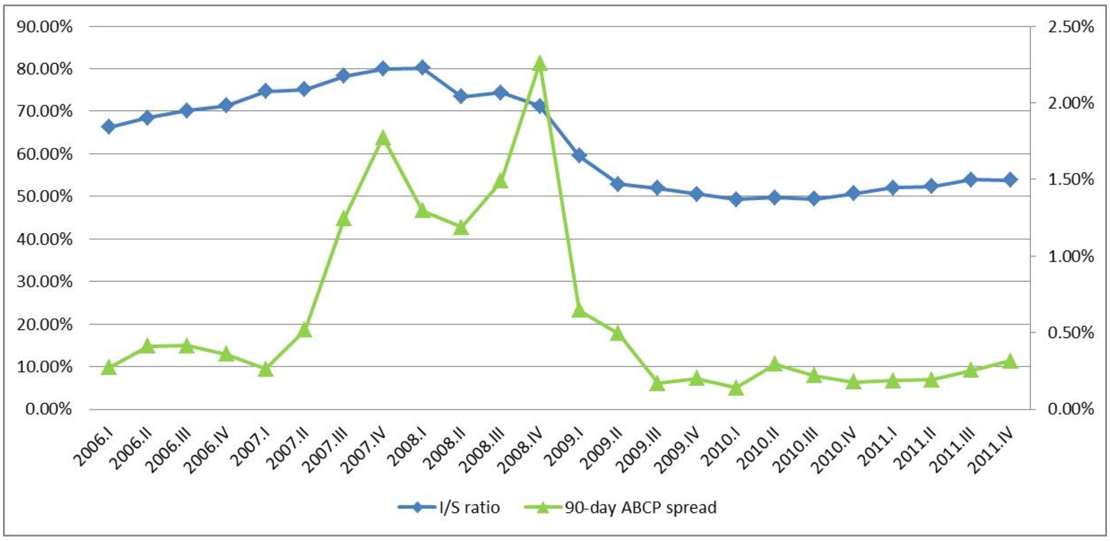


Figure 1: Investment-saving ratio and collateralized lending in US

Notes: Figure 1 portrays the investment-saving ratio (I/S ratio) and the behavior of collateralized lending. The I/S ratio is the ratio of private nonresidential fixed investment to the gross private saving. The ABCP spread measures the difference between the return of the ABCP and the return of a treasury bill with similar maturity. Observe that the I/S ratio began to drop in 2008 and reached the bottom in 2009.IV. Also, even though the ABCP spread reduced largely after the peak in 2008.IV, the I/S ratio did not go back to the level before the financial crisis.

Source: Private nonresidential fixed investment and gross private saving: Federal Reserve Bank of St. Louis-Economic data; ABCP: Federal Reserve Bank-Business Finance Data.

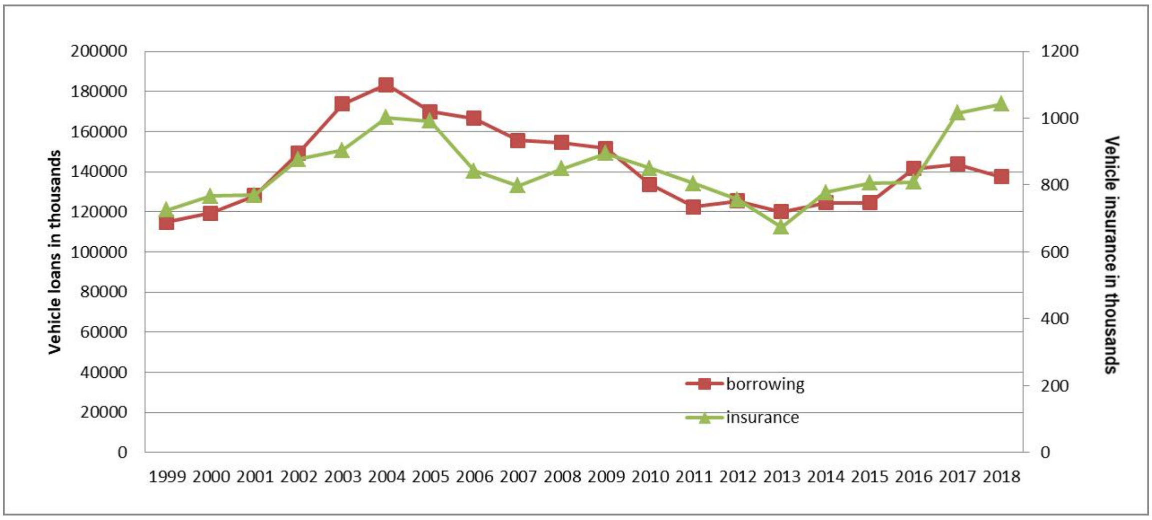


Figure 2: Vehicle loans and insurance in US

Notes: Figure 2 shows the accompanying behavior of vehicle loans and insurance. The vehicles are new when acquired. In the US, the vehicle insurance is compulsory in the most of states and the owners are required to buy more insurance when borrowing to buy the motor vehicle.

Source: Consumer expenditure survey: Public-use micro-data collected by United States Department of Labor.

in the event that collateral becomes worthless in the bust state, and thus, default is inevitable. I define financial innovation[2] as *collateral hedging*. It is important to note that financial innovation is costly. For example, the sellers of CPI, the insurance company, need to evaluate the riskiness of small businesses since the credit-rating scores are not available. Thus, they have to pay a cost in terms of output, that can be either sunk or increasing with the trade volume of CPI, to verify the credit score of the respective business.

In this paper, I consider two distinct economies. The first one is defined as the L-economy–the Leverage economy–where there are no CPI contracts. I assume that parameters are consistent with default in the bust state. Optimists borrow only if they pledge a fraction of their investment as collateral to both moderates and pessimists. The second economy is defined as the CPI-economy, where CPI contracts are traded between different agents. In particular, CPI contracts are traded only between optimists and moderates, because the latter type "quotes" lower prices relative to pessimists, and hence, optimists will always go for cheaper contracts in equilibrium.

I characterize the effectiveness of macro-prudential policies towards avoiding the defaults in the bust state and increasing social welfare. The results show that when belief disagreement (defined appropriately below) is extreme, then financial innovation is not effective, unless it is not costly, and macro-prudential interventions call for higher collateral requirements on capital; while under moderate levels of disagreement, increasing collateral requirements on capital is not effective, and policy interventions call for financial innovation, in the form of CPI contracts. A related interesting question is whether macro-prudential perturbations on the collateral requirements can lead to a Pareto-improvement. There exists following trade-off, on the one hand, tighter collateral regulations lowers welfare through tighter collateral constraints; on the other hand, tightening collateral regulations increases welfare through effects arising from heterogeneous beliefs. The first effect calls for new cheaper collateral, like CPI, while the second effect is reinforced with the introduction of CPI. The results show that introducing CPI contracts induces a Pareto improvement, only if the financial innovation is not too costly.

1.1 Related literature

This model follows the collateral general equilibrium model introduced by Geanakoplos (2003), where endogenous haircuts and equilibrium leverage determined by heterogeneous agents. Fostel and Geanakoplos (2016) used this model to characterize the effects of the financial innovation, namely credit default swaps, on investment. Araujo, Kubler, and Schommer (2012) showed how restricting the sets of tradable assets by regulating collateral requirements induces welfare improvement. Similarly, this paper is introducing collateral hedging to show how collateral regulations can improve social welfare. In addition, Simsek (2013) extends the model of Geanakoplos (2003) with a flexible specification of agents' beliefs. They show how the tightness of collateral constraints are determined by belief disagreement. This argument motivates this research to investigate how heterogeneous beliefs influence the effectiveness of macro-prudential policies on collateral requirements.

Another important strand of the literature is based on Kiyotaki and Moore (1997). In this line of research, the leverage is exogenously fixed, and borrowing with collateral enhances the volatility of asset prices (Bianchi & Mendoza, 2018). Macro-prudential policies aim to limit aggregate risk and reduce macro-economic costs of systemic crises, and to address negative externalities in the financial

[2] Fostel and Geanakoplos (2016) define financial innovation as new types of promises backed by collateral, or the use of new types of collateral.

system (Galati & Moessner, 2013). For example, Rubio and Carrasco-Gallego (2014) proposed a macro-prudential rule for the loan-to-value (LTV) cap which responds to credit growth and the Taylor rule for monetary policy. Mendoza and Bianchi (2011) and Farhi and Werning (2016) suggested macro-prudential polices by levying taxes on debt during the credit boom to mitigate the collateral friction in the downturn. However, some papers pointed out that tightening the collateral constraints is not Pareto-improving (Campbell & Hercowitz, 2009; Lambertini, Mendicino, & Punzi, 2013). This paper characterizes the conditions for the aforementioned macro-prudential policies to induce Pareto-improvement and shows how the effectiveness of the collateral regulation is determined by belief disagreement among agents.

This paper is also related to another strand of literature concerning the plausibility of belief disagreement in welfare analysis. Belief distortions are a key source of heterogeneous beliefs and their presence prompt welfare concerns (Brunnermeier, 2014). Belief distortions arise from psychological biases, such as limited attention and overconfidence (Hirshleifer, 2001). These biases induce agents to react differently to information. In particular, overconfidence causes agents to overreact to certain signals. For example, optimists who focus on their technology opportunities borrow to invest in capital. In contrast, pessimistic agents may be reluctant to lend due to the lack of confidence about technology. As a result, agents who may be unaware of their belief distortions may hurt their own and others' welfare (Brunnermeier, 2014). These arguments motivate my research on how policy interventions mitigate the frictions induced by belief disagreement.

Finally, this paper considers implement-ability of the macro-prudential policy through collateral hedging. Calomiris, Larrain, Liberti, and Sturgess (2017) proposed that creditors in emerging countries are reluctant to accept non real estate assets as collateral because of the limitation in their legal system, while in the US, more than half of collateralized loans issued by small and medium-size business are backed by non-real estate assets. Thus, the strong collateral laws are necessary in the implementation. Moreover, financial innovation is costly (Allen & Gale, 1999). Silber (1983) posited that searching and designing new contracts is the expense of contract issuers. Thus, the financial innovation cost influences whether to implement the macro-prudential policy through collateral hedging. If the financial innovation provides cheaper collateral, it reduces costs of adhering to collateral constraints.

Outline The remained is organized as follows. Section 2 presents the model environment. Section 3 presents portfolio choices of each agent and shows numerical examples to compare the effects of previous macro-prudential policies on welfare in various situations. Section 4 examines the mechanism and characterizes the conditions for Pareto-improvement. Section 5 presents robustness results. Section 6 concludes. Appendices A-F present supplemental material.

2 The Model

2.1 Basic Environment

The economy lasts for two periods, $t = 0, 1$ and has a single consumption good. The uncertainty is characterized by a binomial tree of states $s \in S$, $S = \{0, H, L\}$, where H represents the boom state and L represents the bust state. This paper denotes the time of s by $t(s)$, so $t(0) = 0$ and $t(s) = 1$, $\forall s \in S_1$, the set of terminal nodes of S. Agents i's endowment of the consumption good is represented by e_s^i in each state $s \in S$. The endowments e_0^i enable agents to lend or borrow. Agents transfer their wealth from date 0 to date 1 by investing either financial contracts (q) or capital (k) which deliver returns at date 1. In date 0, the shock to the economy is uncertain. In date 1, the uncertainty is realized by the

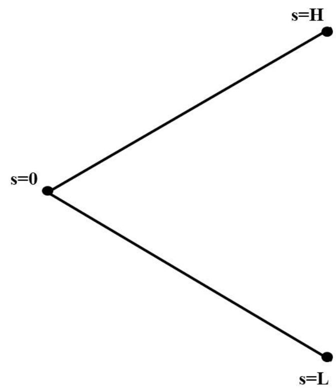


Figure 3: The binomial tree

endowments and the dividends of capital investment. The consumption good is numeraire, and its price is normalized at 1.

Each agent i is characterized by a utility u^i, a discount factor β and different subjective probabilities $\pi_s^i, s \in S_1$. I assume that agents are risk-averse. In each state $s \in S$, the utility function is monotonic, differentiable and strictly concave. The expected utility to agent i is:

$$u^i = u_0^i(c_0^i) + \beta \sum_{s \in S_1} \pi_s^i u_s^i(c_s^i). \tag{1}$$

Suppose agents have identical wealth, utilities and discount rates, but differ in their beliefs. This paper assumes that beliefs are common knowledge among agents. Suppose the correct possibility π_s of the future states is unobservable. Three types of agents, *optimists*, *moderates* and *pessimists* are denoted by $i = 1, i = 2$ and $i = 3$ respectively. They observe some information about π_s and know each others' beliefs at date 0. This means three types of agents agree to disagree. This paper denotes Π to be the given distribution of the heterogeneous beliefs on the bust state $(\pi_L^1, \pi_L^2, \pi_L^3)$, which is discrete and positive. Let σ denote standard derivation corresponding to the beliefs of three types of agents. A larger σ means agents have more distinctive beliefs on the bust state from each other.

Agents 1 act as firms in this model. They invest in capital and provide collateral for borrowing. At date 0, they get access to technology and transform $\alpha > 0$ units of consumption goods into one unit of capital investment. Production investment pays dividends r_s^k of consumption good in each state $s \in S_1$. I assume that $0 < r_L^k < 1 < r_H^k$. Agents 2 and 3 do not invest in capital because of technology barriers, so at date 0, they only lend to agents 1. Finally, three types of agents are necessary to speak about the effect of the introduction of CPI contracts on welfare. Below I will provide more details about that when I discuss portfolio choices in the CPI-economy.

2.2 Financial Contracts

The core part of this research includes financial contracts and collateral. The definition of financial contracts is an agreement including a promise and the collateral backing it and the definition of financial innovation is new types of promises backed by collateral, or the use of new types of collateral. There is no doubt that the payment of collateral depends on the future state of nature. As the collateral dividends are independent from the promise size and other decisions of contract sellers, the consideration of hidden effort is eliminated. Also, I assume that the collateral used to secure one contract cannot back other contracts.

The price of financial contract ($q \in Q$) is m^q and the amount of contracts traded is θ^i_q. A positive θ^i_q indicates that agents i buy contracts q or saving $\theta^i_q m^q$, while a negative θ^i_q indicates that agents i sell contracts q or raise $\mid \theta^i_q \mid m^q$ for financing their investment.

A loan contract ($q = b \in Q^b$) promises 1 unit of consumption goods, backed by a amount of collateral $\boldsymbol{x}^b$ which has to satisfy the collateral requirement $\boldsymbol{C}^b \geq 0$. $\boldsymbol{C}^b$ units of collateral can be pledged for selling one unit of financial contracts. Since the return function of collateral required in the contracts is the same for each agent, $\boldsymbol{F}^i_s(\boldsymbol{C}^b) = \boldsymbol{F}^{i'}_s(\boldsymbol{C}^b)$, there is no adverse selection problem. The contract sellers lose the return of their collateral if they do not honor their promise. Thus, the delivery of loan contracts (r^b_s) at date 1 is

$$min\{1, \boldsymbol{F}^i_s(\boldsymbol{C}^b)\}.$$

The loan contract is priced at m^b and the traded amount is θ^i_b. Each loan contract b has a distinct collateral requirement $\boldsymbol{C}^b = (\phi)$, where ϕ is the collateral requirement on capital.

The CPI contracts ($q = j \in Q^j$) is a derivative contract and its payoffs depend on the actual payoff of loan contracts. As a new type of collateral, it provides additional collateral dividends for loan contracts. In the economy with CPI contracts, $\boldsymbol{C}^b = (\phi, h)$, where h is the collateral requirement on CPI contracts. The collateral $\boldsymbol{x}^b = (k, \theta_j)$. I assume that the return of CPI contracts r^j_s is $(r^j_H, r^j_L) \equiv (0, 1 - \phi r^k_L)$. It is priced at m^j, and the traded amount is θ^i_j. The cost of innovating one unit of CPI is ε. Thus, the cost function is $T(\theta^i_j) = 1_{\{\theta^i_j < 0\}} \varepsilon \theta^i_j$. Most importantly, only capital investors can buy CPI contracts.

In this paper, the interventions are through a loan contract term, namely collateral requirements $\boldsymbol{C}^b$. In the L-economy, only capital can be used as collateral to issue promise, $\boldsymbol{x}^b = (k)$. When $C^b = \phi = 1/r^k_L$, $F^i_L(C^b) = \phi r^k_L = 1$. Increasing collateral requirements will not improve r^b_L, so the collateral regulation manipulates the collateral requirement in the interval $(0, 1/r^k_L]$. In the CPI-economy, the collateral of loan contracts includes both capital and CPI contracts, $\boldsymbol{x}^b = (k, \theta_j)$. Thus, when $h = 1$, $\boldsymbol{F}^i_L(\boldsymbol{C}^b) = \phi r^k_L + h r^j_L = 1$. Hence, the interval of h is $[0, 1]$.

2.3 Budget set

Given commodity price $p_s = 1$ and financial contract prices $((m^q)_{q \in Q})$, agents i decides commodities c_s , for each $s \in S$, investment k and contracts trades θ_q. It is assumed that the expectation of price level is equal to the current price level. This means that the default of loan contracts only depends on the worth difference between promise and collateral, and is independent of the price level. At time 0, maximize utility (1) subject to the budget set defined by

$$\begin{aligned} B^i(m^q) = \{ & (c_s, k, \theta_q) : \\ & (c_0 + \alpha k - e_0) + T(\theta^i_j) + \sum_{q \in Q} \theta_q m^q \leq 0 \\ & c_s - e_s \leq r^k_s k + \sum_{q \in Q} \theta_q r^q_s, \forall s \in S_1 \\ & \sum_{b \in Q^b} max(0, -\theta_b) \boldsymbol{C}^b \leq \boldsymbol{x}^b. \end{aligned}$$

The first inequality requires that money from the sales of contracts to finance money spent on consumption, cost of the financial innovation and investment beyond the endowments in state $0.\mathbf{1}_{\{\theta^i_j < 0\}}$ is the indicator function defined on trading CPI contracts takes on the value 1 when $\theta_j^i < 0$ and 0 otherwise. The second inequality requires that the revenue from dividends of production and contracts can finance money spent on consumption beyond endowments. The last constraint requires that agents actually hold as least as much of collateral as financial contracts require them to hold.

2.4 Collateral equilibrium

A *collateral equilibrium* is a collection of prices, consumption, capital investment and contract trades $((\bar{m^q})_{q\in Q}), (\bar{c_s}^i, \bar{k}, \bar{\theta_q}^i)_{q\in Q}))$, such that

$$\sum_{i=1}^{3} (\bar{c_0}^i + T(\bar{\theta_j}^i) - e_0^i) + \alpha\bar{k} = 0 \tag{2}$$

$$\sum_{i=1}^{3} (\bar{c_s}^i - e_s^i) - r_s^k\bar{k} = 0, \forall s \in S_T \tag{3}$$

$$\sum_{i=1}^{3} w^i\bar{\theta_q}^i = 0, \forall q \in Q \tag{4}$$

$$(\bar{c_s}^i, \bar{k}, \bar{\theta_q}^i) \in B^i(\bar{m^q}), \forall i \tag{5}$$

$$(\bar{c_s}, \bar{k}, \bar{\theta_q}) \in B^i(\bar{m^q}) \Rightarrow U^i(\bar{c_s}) \leq U^i(\bar{c_s}^i), \forall i \tag{6}$$

The first three mean that in the equilibrium markets for consumption goods and contracts in each state clear. The equations (5) and (6) describe that agents optimize their utility subject to their budget sets. As shown in Geanakoplos and Zame (2014), the collateral equilibrium always exists under these assumptions.

I will show how agents design their portfolio in the equilibrium of the L-economy and CPI-economy in the Section 3.1.

2.5 Policy target indicators

As default induces failure of repayment, investment in loans is risky. If collateral generates more dividends in the bust state, the return of loans is less influenced by future states. Loan buyers lose less. This paper focuses on mitigating default friction and chooses r_L^b as the macro-prudential policy target. Thus, the return of loans in the state L

$$r_L^b = \frac{\boldsymbol{F}_L^1(\boldsymbol{C}^b) - m^b}{m^b} = \frac{(\phi r_L^k + hr_L^j) - m^b}{m^b}.$$

$r_L^b \in [-1, (1/m^b - 1)]$. If r_L^b is higher, the loss is less. When $R_L^b < 0$, buying loan contracts leads to a negative return in the bust state. By contrast, when $r_L^b > 0$, lenders receive a positive return. r_L^b increases with a rising ϕ or h, and a declining m^b. When $r_L^b = 0$, investment in loans does not lose any principal value spent. In practice, corporations apply to banks for mortgage loans. If corporations' default induces fewer loan loss provisions [3], the bank system is more resolute (Olszak, Roszkowska, & Kowalska, 2018).

Moreover, it is more intuitive to consider haircuts. This paper studies whether increasing haircuts improves social welfare by mitigating default friction. The haircut is the complement of the LTV ratio

[3] A loan loss provision is an expense set aside as an allowance for uncollected loans and loan repayments.

and the definition of the LTV is a ratio of the total borrowing to the down-payment of capital investment (Geanakoplos & Zame, 2014). Decreasing LTV will enhance the repayment ability of borrowers, so r_L^b increases. In this case, with CPI contracts, optimists invest α units of consumption good to achieve 1 unit of capital investment and buy θ_j^1 units of CPI contracts. These serve as collateral for issuing $\mid \theta_b^1 \mid$ units of loan contracts. Hence, LTV is

$$LTV = \frac{m^b \mid \theta_b^1 \mid}{\alpha k + m^j \theta_j^1}.$$

Also, the haircut is

$$Haircut = 1 - LTV = \frac{\alpha k + m^j \theta_j^1 - m^b \mid \theta_b^1 \mid}{\alpha k + m^j \theta_j^1}.$$

Since agents 1 should pay a fraction of capital investment, $Haircut \in [0, 1)$.

3 Properties of Equilibrium

This section studies the properties of equilibrium influenced by increasing collateral constraints. Firstly, I complete the specification of the model and describe the equilibrium of the L-economy and the CPI-economy.[4] Secondly, this paper presents numerical examples of equilibrium and compares the effectiveness of macro-prudential policy tools, which aims to increase r_L^b. The results motivate the propositions that follow.

3.1 Portfolio choice

3.1.1 L-economy

In the L-economy, without the financial innovation, one unit of loan contract is backed by the income of ϕ units of capital, where $\phi \in [1, 1/r_L^k]$. The delivery is the promised 1 in the boom state, while in the bust state the delivery is ϕr_L^k. Agents 1 borrow by issuing loan contracts, to invest in production. Their portfolio is $\{k, \theta_b^1\}$ where $k > 0, \theta_b^i < 0$. In the equilibrium, $k = -\phi\theta_b^1$, so $Haircut = 1 - m^b/(\phi\alpha)$ and $r_L^b = (\phi r_L^k/m^b) - 1$. Since both indicators are only determined by m^b, $Haircut$ moves monotonically with r_L^b. Agents 2 and 3 lend to agents 1 by purchasing loan contracts. Thus, loan contract buyers whose portfolio is $\{\theta_b^i\}$, where $\theta_b^i > 0$.

Table 1 shows portfolio choices in the L-economy.

Table 1: Portfolio choices in the L-economy

Type	Portfolio choices
Optimists $i = 1$	Buy capital and Sell loans
Moderates $i = 2$	Buy loans
Pessimists $i = 3$	Buy loans

3.1.2 CPI-economy

A Collateral Protection Insurance (CPI) contract which provides insurance on collateral value is issued by lenders. It pays 0 in the boom state and pays $(1 - \phi r_L^k)$ in the bust state. CPI contracts serve as part of collateral for backing loan contracts. Thus, the delivery in the bust state is $(\phi r_L^k + h r_L^j)$.

[4]The equilibrium of the first best case is in the Appendix E, and the results achieved from comparing the equilibrium properties are the same with Fostel and Geanakoplos (2016)

In the CPI-economy, the collateral of one unit of loan contract is the income of ϕ units of capital and h units of CPI contracts. Agents 1 are required to take long position of capital and CPI contracts for borrowing. Their portfolio is $\{k, \theta^1_j, \theta^1_b\}$ where $k > 0, \theta^1_j > 0, \theta^1_b < 0$. Hence, in the equilibrium, $Haircut = 1 - m^b/(\phi\alpha + hm^j)$ and $r^b_L = (\phi r^k_L + hr^j_L)/m^b - 1$. Buying more CPI contracts not only increases the denominator of $Haircut$, but also increases the loan price. Thus, higher h may lead to a lower $haircut$. As a result, $Haircut$ may not move monotonically with r^b_L. It implies that a decrease of haircuts may not reduce the repayment to lenders in the bust state. Additionally, because the return from CPI contracts is used for the repayment of loan contracts, it cannot improve the situation of agents 1 in the bust state. Thus, borrowing more with less spending on collateral is the only motivation of agents 1 to buy CPI contracts.

Agents 2 and 3 lend to agents 1 by purchasing loan contracts. Since agents 3 require more compensation for bearing higher default risk based on their expectation than agents 2 do, agents 1 will only buy CPI contracts from agents 2 who always sell CPI contracts at a lower price than agents 3. Thus, agents 2 choose to take more risk by selling CPI contracts at date 0 and innovate. Their portfolio is $\{\theta^2_j, \theta^2_b\}$, where $\theta^2_j < 0, \theta^2_b > 0$. Moreover, CPI purchasing cannot be separate from other collateral investment. Agents 3 cannot buy CPI contracts because they do not invest in capital. Thus, agents 3 are initially constrained from trading in the CPI contracts. Their portfolio is $\{\theta^3_b\}$, where $\theta^3_b > 0$. In addition, CPI contracts are introduced to increase r^b_L. If there are only two types of agents, lenders are the sellers of CPI and the buyers of loans. The increase of r^b_L they receive is paid back to borrowers in the form of CPI payoff. Thus, the setting with three types of agents is necessary, because agents 3 who do not sell CPI receive more in the bust state.

Table 2 shows portfolio choices in the CPI-economy

Table 2: Portfolio choices in the CPI-economy

Type	Portfolio choices
Optimists $i = 1$	Buy capital, CPI and Sell loans
Moderates $i = 2$	Buy loans and Sell CPI
Pessimists $i = 3$	Buy loans

3.2 Numerical examples

In this section, I present three simulations to compare the effects of the macro-prudential policies in the environment with varying initial collateral requirements ($\bar{\phi}$) and belief disagreement (σ). Specifically, the policy tools are increasing the collateral requirement on capital by $\Delta\phi$ and increasing the collateral requirement on CPI contracts by Δh. The first simulation is one example to compare the equilibrium affected by previous policy interventions with the equilibrium in the benchmark case. The second and third simulations examine the Simulation 1 result in varying environment. The remained is organized as follows. **3.2.1** illustrates the result of Simulation 1. **3.2.2** shows the ABCP spread and I/S ratio in the benchmark cases captured by different $\bar{\phi}$ and σ. Simulation 2 keeps belief disagreement level

Table 3: Fundamental values

	Value		Value		Value
e^i_0	3	α	1.5	π^2_L	0.5
e^i_H	3	r^k_H	3.4	β	1
e^i_L	2	r^k_L	0.8	w^i	0.33

fixed and varies initial collateral requirements, while Simulation 3 assumes the fixed initial collateral requirement and studies the varying belief disagreement of the economy. **3.2.3** compares the effect of two macro-prudential policies tools on welfare, with the policy target fixed. Also, the results show the trigger financial innovation cost level of choosing Δh **3.2.4** and **3.2.5** provide the capital investment and haircut changes induced by previous macro-prudential policies.

In the following sections, I choose $r_L^b = 0$ as the policy target. In the L-economy, the policy tool is increasing the collateral requirement on capital by $\Delta\phi$, while in the CPI-economy, with the financial innovation provides a new policy tool, namely increasing the collateral requirement on CPI contracts by Δh. The three simulations are based on the fundamental characteristics in the Table 3. Also, assume that the specific utility function is $u_s^i = log(c_s^i)$, which is only applied in this subsection. Additionally, to simplify the belief distribution, I suppose that skewness is zero and that π_L^2 is identical in each distribution. Thus, beliefs are given by $\pi_L^1 = \pi_L^2 - \sigma$, π_L^2 and $\pi_L^3 = \pi_L^2 + \sigma$. The each type of agents' belief is ranked as $\pi_L^1 < \pi_L^2 < \pi_L^3$. Notice that this study does not change fundamentals such as utility functions, endowments, capital returns and asset payoffs in the three simulations.

Social welfare V is defined by the weighted sum $\sum_{i=1}^{3} w^i u^i$, where u^i is the expected utility of risk-averse agents i. Suppose that the weight of agents i is the same. Also, V_0 stands for the welfare in the initial equilibrium. V_ϕ and V_h represent social welfare after implementing increasing ϕ and increasing h respectively with the policy target achieved.

3.2.1 An example

Simulation 1 provides a direct comparison of three cases with the following additional parameter values: the benchmark collateral requirements are $\boldsymbol{C}^b = \bar{\phi} = 0.7$, the level of belief disagreement is $\sigma = 0.15$, and the financial innovation cost is $\varepsilon = 0.02$ and the policy target is $r_L^b = 0$. As $r_L^b = (\phi r_L^k + h r_L^j)/m^b - 1$, the r_L^b is increased by a rising ϕ, or a rising h or a declining m^b. The initial collateral requirement in the benchmark case is $\boldsymbol{C}^b = (0.7, 0)$. The reason why assume $\bar{\phi} = 0.7$ is that in the equilibrium, $r_L^b < 0$ calls for increasing collateral requirements.

The results shown in the Table 4 are calculated through solving the maximum problems in the Appendix A and B. The first column shows the equilibrium of the benchmark case, and the second and third columns illustrate the new equilibrium after implementing the macro-prudential policies $\Delta\phi = 0.3967$ and $\Delta h = 0.3654$ respectively. Though both policies drive the price of loan contracts up, the collateral dividends $\boldsymbol{F}_L^1(\boldsymbol{C}^b)$ increases even more. Moreover, the capital investment decreases after

Table 4: Simulation 1: Effects of two macro-prudential policies

	Benchmark	$\Delta\phi = 0.3967$	$\Delta h = 0.3654$
r_L^b	-0.1068	0	0
m^b	0.6270	0.8774	0.7208
k	1.0332	0.9576	1.0230
$Haircut$	0.4029	0.4667	0.3699
u^1	2.1588	2.1824	2.1587
u^2	2.0309	2.0135	2.0291
u^3	1.9653	1.9585	1.9693
V	2.0516	2.0515	2.0523

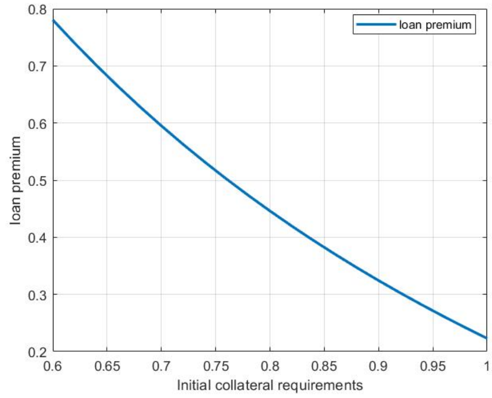


Figure 4: Loan premium for varying $\bar{\phi}$ when $\sigma = 0.15$

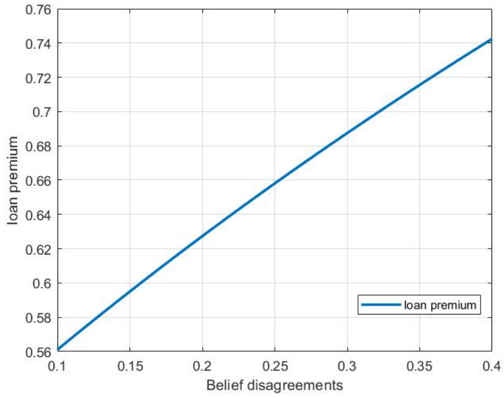


Figure 5: Loan premium for varying σ when $\bar{\phi} = 0.7$

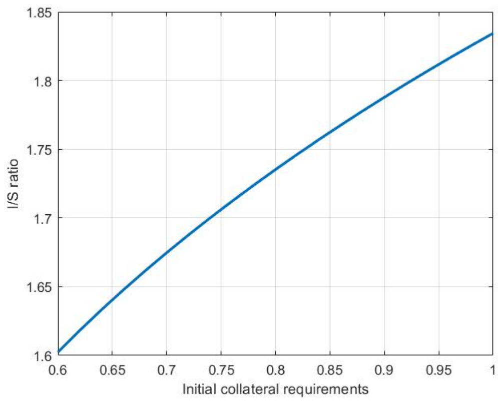


Figure 6: I/S ratio for varying $\bar{\phi}$ when $\sigma = 0.15$

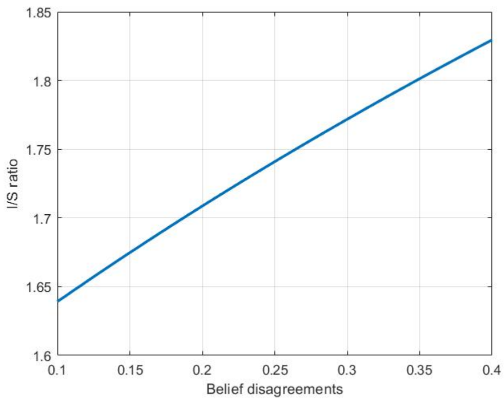


Figure 7: I/S ratio for varying σ when $\bar{\phi} = 0.7$

tightening the collateral constraints. The negative effect of $\Delta\phi$ is $k_\phi - k_0 = 0.9576 - 1.0332 = -0.0756$ and the negative effect of Δh is $k_h - k_0 = 1.0230 - 1.0332 = -0.0102$. Additionally, social welfare in the benchmark case $V_0 = 2.0516$ is decreased to $V_\phi = 2.0515$ by the policy $\Delta\phi$. Since the available financial contracts in the L-economy is limited, the collateral requirement as a contract term determines the risk-sharing in this economy. This negative effect on utilities of agents 2 and 3 reflects that eliminating the loss of default is not beneficial for most of agents who are willing to take risk. By contrast, the policy Δh increases social welfare to $V_h = 2.0523$. However, $Haircut_h$ is higher than $Haircut_0$. This infers that higher haircut does not induce a decline of social welfare. The reason of the welfare improvement is that the CPI contracts provide cheaper collateral and enhance risk-sharing. The result of u^2 and u^3 implies that agents 2 and 3 are willing to bear some risk. Last, as for the welfare of each type of agents, welfare of agents 2 decreases after implementing both policy tools. This motivates the discussion of the Pareto-improving intervention in the section 4.3.

3.2.2 Loan premium and I/S ratio

This section shows that the model explains how the initial collateral requirements and belief disagreement affect the fluctuation of loan premium and I/S ratio. The loan premium defined as the promised return over the loan price $1/m^b$ is the proxy of ABCP spread. Figure 5 shows that during the crisis, the belief disagreement is extreme and the loan premium is highest. It infers that if loan contract buyers hold much more pessimistic beliefs than loan contract sellers do, the promises of borrowers are valued much less by lenders. This argument supports that before the financial crisis, the increase of the ABCP spread results from the extreme belief disagreement. Figure 4 demonstrates that increasing collateral requirements can drive the loan premium down, with a fixed belief disagreement level. Also, the figure 5 shows that when the belief disagreement is reduced, the loan premium goes down. The results point out that the decline of ABCP spread after the financial crisis is induced by tightening collateral requirements and less belief disagreement.

In addition, the I/S ratio is the capital investment over the borrowing, $\alpha k/(-m^b\theta_b^1)$. Since in the equilibrium, $k = -\phi\theta_b^1$, $I/S = \alpha\phi/m^b$. Figure 6 illustrates that during the crisis, when the collateral constraint is loose, the I/S ratio is low. Though raising collateral requirements can increase I/S ratio, the macro-policies which reduce the belief disagreement hold this growth. It is shown in the figure 7 that the I/S ratio decreases with the decline of the belief disagreement when $\bar{\phi}$ is fixed. The result answers the first question in the beginning. Before the recession, the belief disagreement increased, and the I/S ratio went up. In the recession, the low collateral requirement induced the lenders' fear of loss, resulting a poor I/S ratio. After the recession, the I/S ratio recovered slowly because the less belief disagreement call for lower collateral requirement which improves the pledge-ability of collateral. Therefore, the policy makers need to consider the belief disagreement when designing collateral regulations.

3.2.3 Welfare

It is also interesting to study how the sign of the effect on welfare depends on the initial collateral requirement, belief disagreement and financial innovation cost in the CPI-economy. Simulation 2 and 3 extend simulation 1, and the policy tools $\Delta\phi > 0$ and $\Delta h > 0$ are to achieve $r_L^b = 0$ [5]. Firstly, simulation 2 examines the varying initial collateral requirements $\bar{\boldsymbol{C}}^b = (\bar{\phi})$. The results are illustrated in the figure 8. If the borrowing constraint is initially tight, increasing the collateral requirement on capital hurts social welfare. On the other hand, intervening the benchmark case with a loose collateral constraint induces more welfare improvement. Figure 8 also displays that the necessity of the financial innovation depends on its cost. To figure out the trigger cost ε_{max}, I suppose a threshold of $\bar{\phi}$ is represented by $\bar{\phi}_{max}$. If $\bar{\phi} > \bar{\phi}_{max}$, the policy tools drive social welfare down. The solid line crosses the horizontal axis at $\bar{\phi}^{\phi}_{max} = 0.6826$. When issuing one unit CPI costs $\varepsilon \geq \varepsilon_{max} = 0.0313$, $\bar{\phi}^{\phi}_{max}$ is higher than $\bar{\phi}^{h}_{max}$. Thus, the financial innovation is redundant for designing macro-prudential policies. If ε is less than ε_{max}, the dash line shifts upward and $\bar{\phi}^{h}_{max} > \bar{\phi}^{\phi}_{max}$. The policy tool Δh should replace the other policy tool $\Delta\phi$ for achieving $r_L^b = 0$, when $\bar{\phi}^{h}_{max} > \bar{\phi} > \bar{\phi}^{\phi}_{max}$.

In addition, simulation 3 examines how the effect of policy tools on welfare changes as the level of belief disagreement increases in the L-economy and the CPI-economy. The lines in the figure 9 present that the effects of increasing the collateral requirements on welfare are not always positive. The result supports that identification of heterogeneous beliefs is necessary to the macro-prudential policy design. Previous literature proved that if financial markets are not complete, investors with inaccurate beliefs may survive in the equilibrium (Sandroni, 2000). Hence, the economic analysis should consider heterogeneous prior beliefs (Morris, 1995), because the belief disagreement may not disappear after Bayesian learning in the long run (Acemoglu, Chernozhukov, & Yildiz, 2016).

Figure 9 shows that when σ is close to 0, both policy tools reduce social welfare. This means that the target $r_L^b = 0$ causes an over-tight borrowing constraint, if agents share a similar belief. Lenders are willing to bear some risk, when they believe that borrowers are very likely to keep promises. Suppose

[5] r_L^b is less than 0 in each benchmark case characterized by $\bar{\phi}$ and σ. The result is presented in the Appendix F

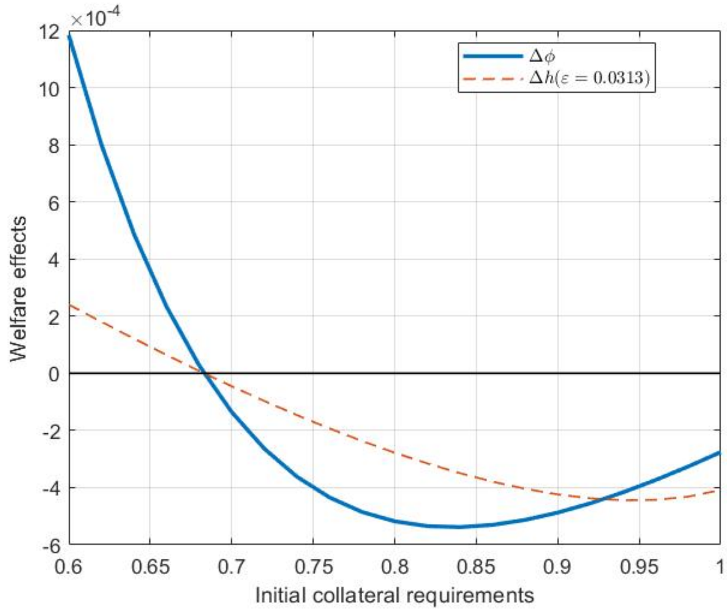


Figure 8: Simulation 2: Welfare effects for varying $\bar{\phi}$

Notes: $\sigma = 0.15$. $\Delta\phi > 0$ and $\Delta h > 0$ is used to achieve $r_L^b = 0$. The solid line describes the effect on social welfare induced by $\Delta\phi > 0$ and the dashed line captures the welfare effects of $\Delta h > 0$. The welfare effects of these polices are $(V_\phi - V_0)$ and $(V_h - V_0)$. Also, the dashed line is characterized by the financial innovation cost $\varepsilon = 0.0313$. If $\bar{\phi} < 0.6$ or $\bar{\phi} > 1$, there is no proper solution with the fundamental values in Table 3.

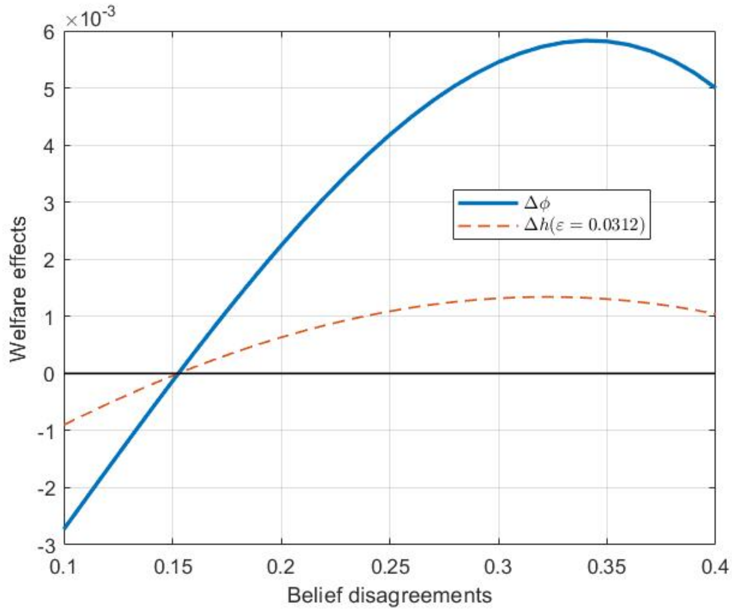


Figure 9: Simulation 3: Welfare effects for varying σ

Notes: $\bar{\phi} = 0.7$. $\Delta\phi > 0$ and $\Delta h > 0$ is used to achieve $r_L^b = 0$. The solid line describes the change of social welfare induced by $\Delta\phi > 0$ and the dashed lines capture the welfare effects of $\Delta h > 0$. The welfare effects of these polices are $(V_\phi - V_0)$ and $(V_h - V_0)$. Also, the dashed line is characterized by the financial innovation cost $\varepsilon = 0.0312$. If $\sigma > 0.4$ or $\sigma < 0.1$, there is no proper solution with the fundamental values in Table 3.

that there is a threshold of belief disagreement level is represented by σ_{min} . In the figure 9, the solid line and the dashed line cross the horizontal axis at σ^{ϕ}_{min} and σ^{h}_{min}. It means that if $\sigma < \sigma_{min}$, the policy tools drive social welfare down. According to fundamental value in Table 3, the threshold of the policy tool $\Delta\phi$ is $\sigma^{\phi}_{min} = 0.1528$. If ε increases to the trigger level, $\varepsilon_{max} = 0.0312$ or even larger, the threshold of the policy tool Δh (σ^{h}_{min}) is higher than σ^{ϕ}_{min}. Therefore, the financial innovation is not proposed. If ε is less than ε_{max}, the dash line moves upward and $\sigma^{h}_{min} < \sigma^{\phi}_{min}$ .The policy tool Δh should replace the other policy tool $\Delta\phi$ for achieving $r^b_L = 0$, when $\sigma^{h}_{min} < \sigma < \sigma^{\phi}_{min}$.

3.2.4 Capital investment

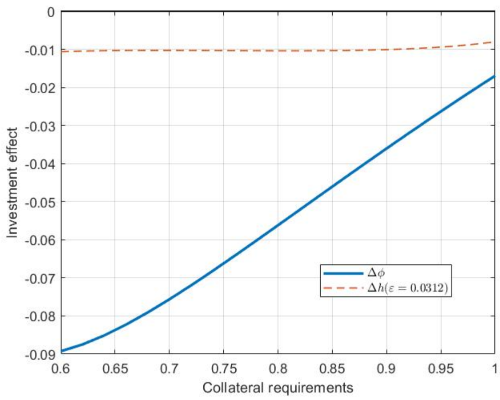


Figure 10: Investment effects for varying $\bar{\phi}$

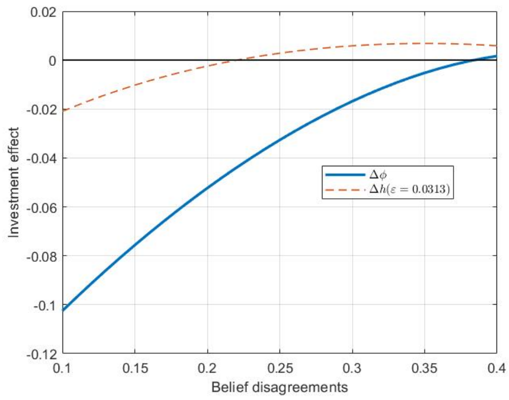


Figure 11: Investment effects for varying σ

Notes: $\Delta\phi > 0$ and $\Delta h > 0$ is used to achieve $r^b_L = 0$. The solid line describes the change of capital investment induced by $\Delta\phi > 0$ and the dashed lines capture the investment effects of $\Delta h > 0$. The investment effects of these polices are $(k_\phi - k_0)$ and $(k_h - k_0)$.

In this section, this paper examines the effects of two policy tools on capital investment. Figure 10 shows that with the belief disagreement fixed at 0.15, the policy tools $\Delta\phi > 0$ or $\Delta h > 0$ both lead capital investment to reduce. Also, $(k_\phi - k_0)$ is larger than $(k_h - k_0)$, especially when the initial collateral requirement is low. It implies that if the collateral regulation induces a larger increase on the cost of achieving collateral, the decline of capital investment is larger. However, the decrease of capital investment does not always result in a negative effect on welfare, according to the figure 7. When the collateral constraint is loose, $\bar{\phi} < \bar{\phi}^{\phi}_{max}$, reducing capital investment eliminates the aggregate risk and improves social welfare.

Furthermore, this paper studies the effects on investment for varying belief disagreement. Figure 11 shows that with $\bar{\phi} = 0.7$, when the policy tools $\Delta\phi > 0$ or $\Delta h > 0$ will raise the capital investment when the belief disagreement is extreme. This means that higher collateral requirements increase the loan payoffs and motivate agents 2 and 3 to lend. In addition, if the heterogeneity of agents' beliefs is lower, a big increase of r^b_L means a larger increase of borrower restrictions on the credit access. An excessive restriction in L-economy can drive the production and social welfare down. The marginal σ^k_ϕ, which means $\Delta\phi > 0$ does not affect the investment, in the L-economy is 0.3840, and in the CPI-economy, $\sigma^k_h = 0.2200$.

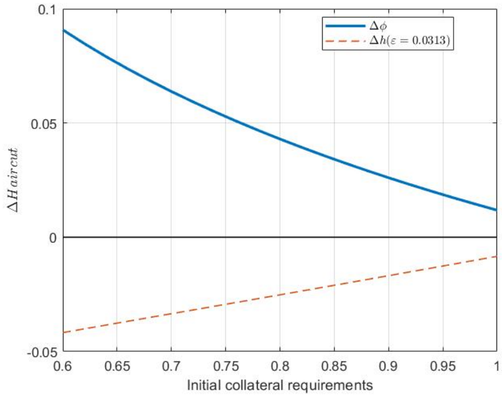


Figure 12: Haircut effects for varying $\bar{\phi}$ when $\sigma = 0.15$

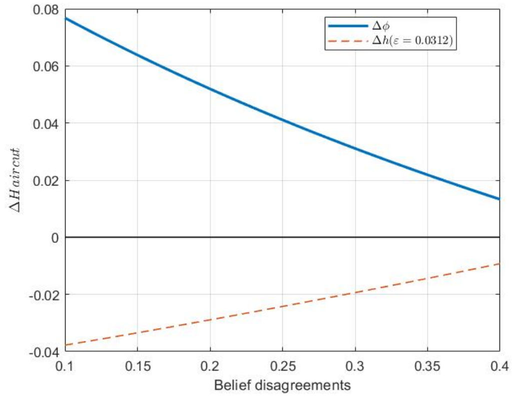


Figure 13: Haircut effects for varying σ when $\bar{\phi} = 0.7$

Notes: $\Delta\phi > 0$ and $\Delta h > 0$ is used to achieve $r_L^b = 0$. $\Delta Haircut$ stands for the $Haircut$ changes of these polices. The solid line captures $(Haircut_\phi - Haircut_0)$. The dashed line captures $(Haircut_h - Haircut_0)$.

3.2.5 Haircuts

In the last part, I show the haircut changes after macro-prudential intervention in the Simulation 2 and 3. Figure 12 and 13 show that $\Delta\phi > 0$ will always induce a positive effect on $Haircut$. Moreover, with $\Delta h > 0$, figure 12 and figure 13 illustrate that $\Delta Haircut$ is always negative. It reveals that the increase of m^b dominates the $Haircut$ changes. According to the results in the figure 8 and 9, this paper proves that a higher haircut does not always improve social welfare. With the introduction of CPI contracts, though increasing h reduces the haircuts, it drives social welfare up. As r_L^b increases, a decrease in haircuts may not weaken the resilience of financial system. Since the financial innovation provides cheaper collateral to borrowers, the non-financial firms are easier to get access to credit which is an important driver of their growth (Volk & Trefalt, 2014).

4 Macro-prudential Policies

The previous section has shown how the macro-prudential tools affect social welfare in the L-economy and the CPI-economy with financial innovation cost. In this section, this paper illustrates the mechanism to explain why the tightness of endogenous borrowing constraints and the financial innovation cost matters. Then, I present the collateral and heterogeneity effects in the model setting and the proposition regarding the Pareto-improving intervention.

4.1 Perturbations on collateral requirements

The figures present that the effects of two macro-prudential tools on welfare are determined by belief disagreement, original collateral requirements and the financial innovation cost. In this section, I will show whether this description always fit from a general perspective.

Figure 8 reflects that the welfare improvement of increasing collateral requirement on capital is weakened by the tightness of collateral constraint in the L-economy. It is captured by the Lagrangian multiplier of the collateral constraint on capital which is represented by $\bar{\mu}^1$. Moreover, figure 8 shows that the heterogeneity of agents' beliefs matters. The different beliefs result in an inequality of the agents' present values of future wealth. The present values $\gamma_s^i = \lambda_s^i/\lambda_0^i, s \in S_T$, where $\lambda_s^i, s \in S$ is the marginal utilities of consumption. Thus, with the assumption $\pi_L^1 < \pi_L^2 < \pi_L^3$, in the incomplete market, the present values in the original equilibrium is ranked as $\bar{\gamma_L}^1 < \bar{\gamma_L}^2 < \bar{\gamma_L}^3$.

In the L-economy, the perturbation $d\phi$ at the initial state, where $d\phi > 0$ induces marginal changes at date 0, $(dc_0^i, dk, d\theta_b^i)$,with $\sum_{i=1}^{3} w^i d\theta_b^i = 0$, and then adjustments of the subsequent equilibrium plans and prices (dc_s^i, dm^b) around the equilibrium $(\bar{c_s}^i, \bar{\theta_b}^i, \bar{m^b})$. Assume $\bar{\boldsymbol{C}}^b = (\bar{\phi})$, where $\bar{\phi} \in (0, 1/r_L^k)$. The complete derivation is in Appendix C.I.[6] The change in social welfare is

$$(\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} |_{\phi=\bar{\phi}})/d\phi = (w^2\bar{\theta_b}^2(\bar{\gamma_L}^2 - \bar{\gamma_L}^1) + w^3\bar{\theta_b}^3(\bar{\gamma_L}^3 - \bar{\gamma_L}^1))r_L^k - (w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3)\frac{\bar{\mu}^1}{\bar{\lambda_0}^1}. \quad (7)$$

Hence, the effect of a positive $d\phi$ on social welfare is determined by the sign of equation (7). Since agents without technology access can only distribute their wealth through investing loan contracts, $\bar{\theta_b}^i > 0, i = \{2, 3\}$. As a result, the discussion mainly focuses on $(\bar{\gamma_L}^i - \bar{\gamma_L}^1), i = \{2, 3\}$ and $\frac{\bar{\mu}^1}{\bar{\lambda_0}^1}$. The former term captures belief disagreement. A negative sign can be induced by the belief distribution with a low σ or a high initial collateral requirement $\bar{\phi}$. If $\frac{\bar{\mu}^1}{\bar{\lambda_0}^1}$ is higher than $r_L^k(\bar{\gamma_L}^2 - \bar{\gamma_L}^1)$, the sign of equation (7) is negative. Less heterogeneity in beliefs reflects that agents 2 and 3 would like to share risk with agents 1 and the high $\bar{\phi}$ sets risk-taking barriers to agents 2 and 3. In this situation, increasing ϕ possibly does harm to the economy. In addition, $\frac{\bar{\mu}^1}{\bar{\lambda_0}^1}$ means the substitution rate between the value of holding capital as collateral and the marginal utility of consumption at the initial date. A higher collateral requirement induces the needs for more collateral to satisfy borrowing needs. Thus, the perturbation $d\phi > 0$ does not always benefit social welfare. The result motivates a proposition:

Proposition 1. *From equation (7) and the discussion above, tightening the collateral constraint is not always welfare-improving. Belief disagreement* (σ) *and the tightness of the initial collateral constraint* ($\bar{\mu}^1$) *determine whether increasing the collateral requirement on capital, namely* $d\phi > 0$ *improves social welfare.*

In the CPI-economy, with the financial innovation, the CPI market exists. The perturbation $dh > 0$ at date 0, induce marginal changes $(dc_0^i, dk, d\theta_b^i, d\theta_j^i)$,with $\sum_{i=1}^{3} w^i d\theta_b^i = 0$ and with $\sum_{i=1}^{3} w^i d\theta_j^i = 0$. The following adjustments are around the equilibrium $(\bar{c_s}^i, \bar{\theta_b}^i, \bar{\theta_j}^i, \bar{m^b}, \bar{m^j})$. Assume $\bar{\boldsymbol{C}}^b = (\bar{\phi}, \bar{h})$, where $\bar{\phi} \in (0, 1/r_L^k)$ and $\bar{h} \in (0, 1]$.[7] The subsequent equilibrium plans and prices are (dc_s^i, dm^b, dm^j).The complete derivation is in Appendix C.II. The change in social welfare is

$$(\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} |_{h=\bar{h}})/dh = w^3\bar{\theta_b}^3 r_L^j(\bar{\gamma_L}^3 - \bar{\gamma_L}^2) + w^1\bar{\theta_b}^1\varepsilon. \quad (8)$$

The positive sign of equation (8) stands for a positive effect of the perturbation $dh > 0$ on welfare. First, I study the sign of the first term. Since $\bar{\theta_b}^3 > 0$, the sign of first term is positive as long as $(\bar{\gamma_L}^3 - \bar{\gamma_L}^2) > 0$. As the pessimists' valuation on CPI contracts is higher than the moderates' valuation, agents 3 are initially constrained from selling CPI contracts. Thus, the effectiveness of this perturbation

[6]The effects of collateral on welfare in the no-default case is explained in the Appendix D.

[7]Because of the financial innovation cost, the derivation function (8) cannot discuss the equilibrium with $\bar{h} = 0$. However, if $\bar{h}$ is infinitely close to zero, with the assumed numbers in the Table 1, the resource allocation and welfare of CPI-economy is almost identical to that of L-economy in the equilibrium. Thus, the discussion of marginal changes induced by dh reflects the general mechanism of the results in the figures 8 and 9.

is determined by the belief disagreement among lenders. If lenders are homogeneous, all of them will sell CPI contracts and the market structure is not improved by the financial innovation. Also, social welfare in the economy with the varying h is the same.

Compared with the perturbation $d\phi > 0$, the perturbation $dh > 0$ is less influenced by the tightness of collateral constraints in the benchmark equilibrium. Firstly, the heterogeneity between agents 1 and others does not influence the effect of dh on welfare. The reason is that the collateral constraint on the CPI enables agents 1 to share risk with agents 2 by buying CPI contracts. Moreover, the $\bar{\mu}^1$ and $\bar{\eta}^1$ relating the value of holding collateral are not shown in the equation (8). Since there is no change in ϕ, $d\phi = 0$, the tightness of collateral constraint on capital remains the same, $\bar{\mu}^1 \times d\phi = 0$. Also, this CPI market clearing ensures that increasing h will not induce the shortage of collateral. Hence, $\bar{\eta}^1$ does not affect social welfare.

Next, I investigate the sign of second term in the equation (8). Since agents 1 borrow to invest in capital ($\bar{\theta_b}^1 < 0$) and $\varepsilon > 0$, the sign of second term is negative. Thus, If the financial innovation cost is higher than the trigger level $\varepsilon_{max} = -w^3\bar{\theta_b}^3 r_L^j(\bar{\gamma_L}^3 - \bar{\gamma_L}^2)/w^1\bar{\theta_b}^1$, the perturbation $dh > 0$ drives social welfare down. The dashed lines in the previous figures also capture ε_{max}. Therefore, this perturbation is beneficial in the economy where the cost is low enough. Then, I can give the second proposition.

Proposition 2. *In the economy with collateral hedging, the financial innovation cost (ε) and belief disagreement among lenders determine whether to perturb the collateral requirement on CPI contracts (h). If $\varepsilon < \varepsilon_{max}$, the perturbation $dh > 0$ improves social welfare, where*

$$\varepsilon_{max} = -\frac{w^3\bar{\theta_b}^3 r_L^j(\bar{\gamma_L}^3 - \bar{\gamma_L}^2)}{w^1\bar{\theta_b}^1}.$$

Proposition 2 also implies that in the economy with a large belief disagreement between agents 2 and 3, the macro-prudential policies via financial innovation is welfare improving though the financial innovation costs much. The reason is that a high collateral requirement on CPI contracts reduces pessimists' loss when agents 1 default and benifits them. The increase of agents 3's utilities compensates the friction induced by the financial innovation cost. This also explains the results in the figures 8 and 9. Firstly, a low initial collateral requirement $\bar{\phi}$ implies a high potential loss of holding loan contract. Since agents 3 are risk-averse, the additional collateral dividends will lead to higher utility if $\bar{\phi}$ is lower. Thus, the friction of the financial innovation cost will not induce a negative effect of the perturbation $dh > 0$ on welfare. Secondly, a high belief disagreement implies that agents 3 are more pessimistic with a fixed belief of agents 2. Thus, an increase of h can improve the utilities of agents 3 more.

4.2 Collateral and heterogeneity effects

Equation (7) shows that there is a trade-off following the perturbation on ϕ. The first term characterizes the heterogeneity effect which increases welfare. Since lenders are more pessimistic, the increase of lenders' utilities by higher loan return in the bust state is higher than the decrease of borrowers' utility. The second term characterizes the collateral effect which decreases welfare. A higher ϕ requires agents 1 to satisfy more consumption at date 0 for investing in collateral. With the initial endowments, agents 1 cannot acquire as much as capital collateral to satisfy their borrowing needs. To mitigate collateral effects, financial innovation, which provides cheaper collateral to agents 1, is compelling.

Equation (8) shows another type of trade-off. The first term characterizes the heterogeneity effect. However, trading the CPI market between agents 2 and 3, whereas eliminates heterogeneity effects caused by the belief disagreement between agents 1 and the others. Welfare is improved because the increase of pessimists' utilities larger than the decrease of moderates utilities. In addition, the second term which drives welfare down is about financial innovation cost. The higher h requires moderates produce more CPI contracts. Thus, a high financial innovation cost makes the macro-prudential policies through financial innovation unnecessary. Intuitively, the results show that the nature of belief disagreement determines whether tightening borrowing constraints is an over-protection, which limits lenders receiving the compensation by sharing aggregate risk. It leads to a decline of social welfare. Therefore, the financial innovation is necessary to enhance risk-sharing.

4.3 Pareto-improving intervention

In this section, I will present the Pareto-improving intervention in the CPI-economy. The table 4 shows that in the CPI-economy, agents 2 are worse off. This is because as CPI sellers, they bear more default risk, however, agents 3 are beneficial from the higher collateral requirement on CPI contracts without any cost. Therefore, the Pareto-improving intervention includes transfers at the initial state, when the collateral requirement on CPI contracts, $dh > 0$ is enforced.

To give the Pareto-improving intervention (dh, dt^i), this study will continue the derivation analysis on the effects of the perturbation,$dh > 0$, on welfare with a condition $\sum_{i=1}^{3} w^i dt^i = 0$ at date 0. Then, the changes on utility of each type are:

$$\frac{du^1}{\bar{\lambda}_0^1} \mid_{h=\bar{h}} = -(\bar{\theta}_b^{\,1} dm^b + \bar{\theta}_j^{\,1} dm^j) + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda}_0^1})\bar{\theta}_b^{\,1} dh + dt^1, \tag{9}$$

$$\frac{du^2}{\bar{\lambda}_0^2} \mid_{h=\bar{h}} = -(\bar{\theta}_b^{\,2} dm^b + \bar{\theta}_j^{\,2} dm^j) + \bar{\gamma_L}^2 r_L^j \bar{\theta}_b^{\,2} dh + dt^2 \tag{10}$$

$$\frac{du^3}{\bar{\lambda}_0^3} \mid_{h=\bar{h}} = -\bar{\theta}_b^{\,3} dm^b + \bar{\gamma_L}^3 r_L^j \bar{\theta}_b^{\,3} dh + dt^3. \tag{11}$$

The first terms in equation (9), (10) and (11) describe the indirect effects arising from the changes in the market clearing prices (dm^b, dm^j). The second terms in equations (9), (10) and (11) are the present value of state L income from buying more units of CPI contracts to each type of agents.

Suppose $\varepsilon < -w^3 \bar{\theta}_b^{\,3} r_L^j (\bar{\gamma_L}^3 - \bar{\gamma_L}^2)/w^1 \bar{\theta}_b^{\,1}$, $\delta = (r_L^j w^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2)\bar{\theta}_b^{\,3} + w^1 \bar{\theta}_b^{\,1} \varepsilon) dh > 0$. Therefore, to make every agent better off, the initial transfers are

$$\begin{aligned}
dt^1 &= -(-\bar{\theta}_b^{\,1} dm^b - \bar{\theta}_j^{\,1} dm^j + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda}_0^1})\bar{\theta}_b^{\,1} dh) + \delta, \\
dt^2 &= -(-\bar{\theta}_b^{\,2} dm^b - \bar{\theta}_j^{\,2} dm^j + \bar{\gamma_L}^2 r_L^j \bar{\theta}_b^{\,2} dh) + \delta, \\
dt^3 &= -(-\bar{\theta}_b^{\,3} dm^b + \bar{\gamma_L}^3 r_L^j \bar{\theta}_b^{\,3} dh) + \delta.
\end{aligned}$$

Therefore,

$$\frac{du^i}{\bar{\lambda}_0^i} \mid_{h=\bar{h}} = \delta > 0.$$

The positive heterogeneity effects are redistributed by the transfers. In addition, the mitigation of collateral effects requires $\varepsilon < -w^3 \bar{\theta}_b^{\,3} r_L^j (\bar{\gamma_L}^3 - \bar{\gamma_L}^2)/w^1 \bar{\theta}_b^{\,1}$. This condition can be reorganized into,

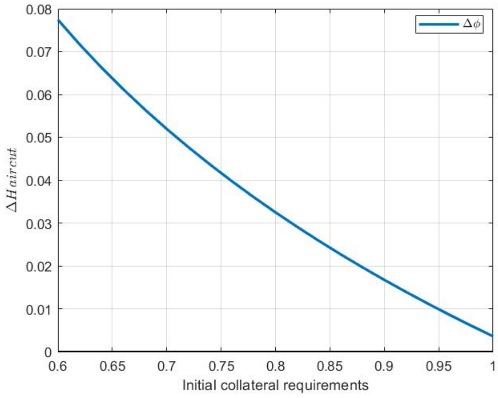


Figure 14: Haircut effects for varying $\bar{\phi}$ when $\sigma = 0.2$

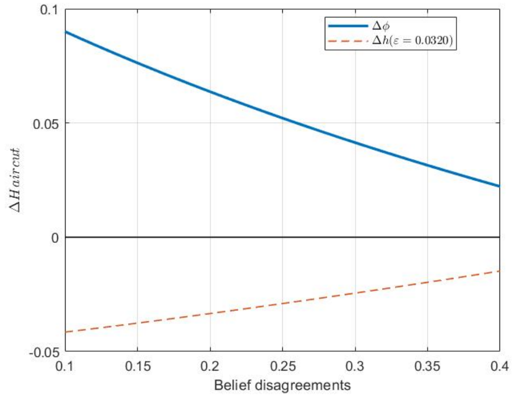


Figure 15: Haircut effects for varying σ when $\bar{\phi} = 0.65$

Notes: $\Delta\phi > 0$ and $\Delta h > 0$ is used to achieve $r_L^b = 0$. $\Delta Haircut$ stands for the $Haircut$ effects of these polices. The solid line captures ($Haircut_\phi - Haircut_0$). The dashed line captures ($Haircut_h - Haircut_0$).

$$w^1 \bar{\theta_j}^1 \varepsilon < w^3 \bar{\theta_b}^3 \bar{h} r_L^j (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) \tag{12}$$

The left-hand side is the total financial innovation cost. The right-hand side shows the net benefits of the society. It equals the benefits of agents 3 minus the loss of agents 2 when h increases. The complete proof of the Pareto-improvement is in Appendix C.III.

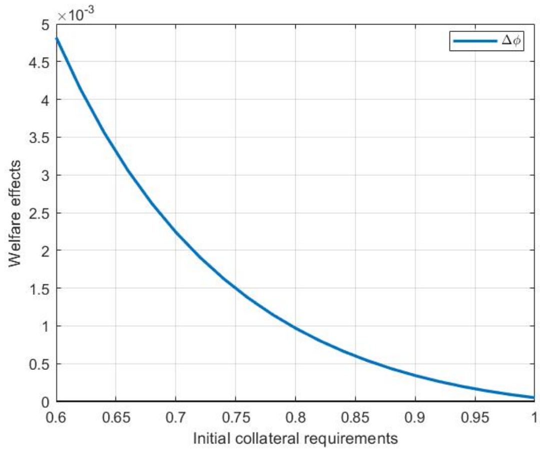
As a result, I can give the third proposition:

Proposition 3. *Given an economy where equation (12) holds, if loan contract buyers have heterogeneous expectations on future cash flow of limited collateral assets, then collateral hedging enhances risk sharing and increases social welfare. Moreover, increasing collateral requirement on CPI contracts, combined with the appropriate transfers at date 0, induces a Pareto improvement.*

5 Robustness Analysis

In the section 3.2.3 and 3.2.4, Simulation 2 fixes the belief disagreement at 0.15 to study the policy effects when initial collateral requirements vary. Simulation 3 fixes the initial collateral requirement at 0.7 to study the policy effects when belief disagreement varies. Figure 8 shows that $\bar{\phi} = 0.7 > \bar{\phi}^\phi_{max}$ and figure 9 displays that $\sigma = 0.15 < \sigma^\phi_{min}$. In order to verify if these two specifications are robust, I do the simulation 2 with $\sigma = 0.2 > \sigma^\phi_{min}$ and the simulation 3 with $\bar{\phi} = 0.65 < \bar{\phi}^\phi_{max}$.

Compared with figure 8, figure 16 illustrates that when belief disagreement is larger than the trigger level, namely $\sigma = 0.2 > \sigma^\phi_{min}$, increasing ϕ always improves social welfare if the initial collateral requirement is no more than 1. Thus, the financial innovation is not necessary. As for the simulation 3, the lines in the figure 17 is less curved than those in the figure 8. It reveals that when $\bar{\phi}$ declines, σ_{min} also decreases. Additionally, the haircut effects shown in the figure 14 and figure 15 are similar to that in the figure 12 and 12. I also test the simulation 2 with σ fixed at 0.1, 0.25, and the simulation 3 with $\bar{\phi}$ fixed at 0.6 and 0.75. The characteristics of lines are similar. In a word, the robustness check generates the same intuitions in the previous sections.



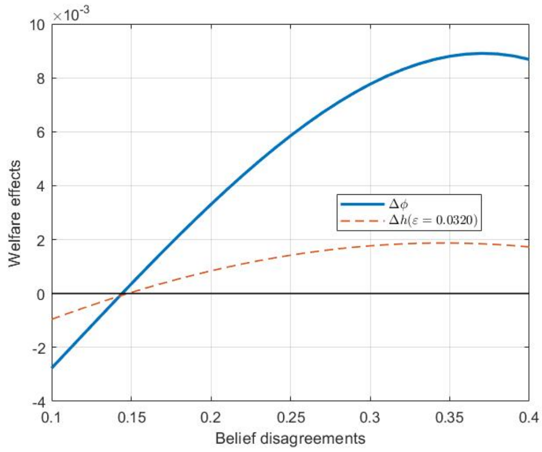
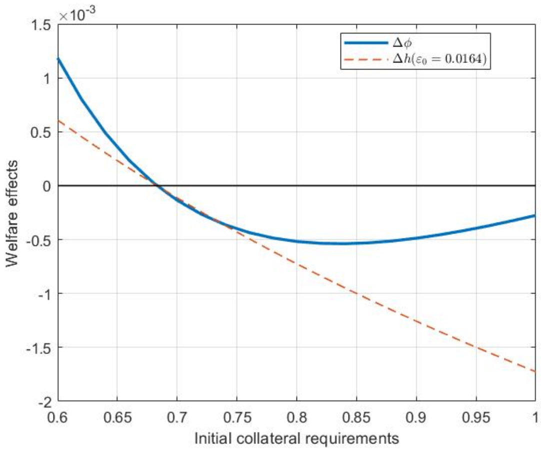


Figure 16: Welfare effects for varying $\bar{\phi}$ at $\sigma = 0.2$

Figure 17: Welfare effects for varying σ at $\bar{\phi} = 0.65$

Notes: $\Delta\phi > 0$ and $\Delta h > 0$ is used to achieve $r_L^b = 0$. The solid line describes the change of social welfare induced by $\Delta\phi > 0$ and the dashed lines capture the welfare effects of $\Delta h > 0$. The welfare effects of these polices are $(V_\phi - V_0)$ and $(V_h - V_0)$. Also, from top to bottom, the dashed line is characterized by the financial innovation cost ε.



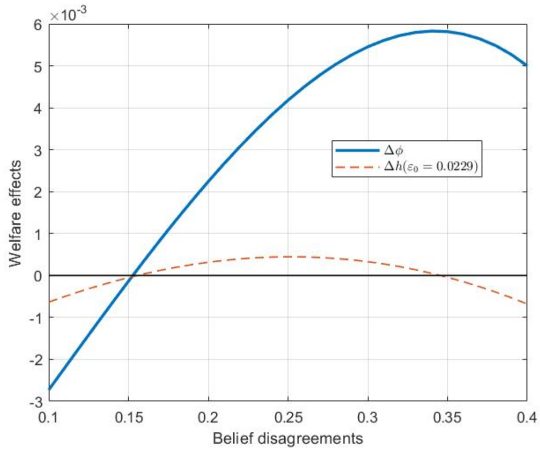


Figure 18: Welfare effects for varying $\bar{\phi}$ at $\sigma = 0.15$

Figure 19: Welfare effects for varying σ at $\bar{\phi} = 0.7$

Notes: $\Delta\phi > 0$ and $\Delta h > 0$ is used to achieve $r_L^b = 0$. The solid line describes the change of social welfare induced by $\Delta\phi > 0$ and the dashed line capture the welfare effects of $\Delta h > 0$. The welfare effects of these polices are $(V_\phi - V_0)$ and $(V_h - V_0)$. Also, from top to bottom, the dashed line is characterized by the financial innovation cost ε_0.

Then, I discuss the assumption that the financial innovation cost depends on the volume of CPI contracts. Now, I consider another specification where the financial innovation induces sunk cost. Note that the equilibrium in the CPI-economy with $h = 0$ is different from that in the L-economy, because the sunk cost reduces the total wealth.

Based on the fundamental values in the Table 3, I study whether initial collateral requirements and belief disagreement influence the effects of the two policies on welfare, conditioning that the cost function is $T(\theta_j^i) = 1_{\{\theta_j^i<0\}}\varepsilon_0$. The results are shown in the figure 18 and figure 19. Figure 18 proves the results in the figure 8 are robust. However, figure 19 displays that besides the threshold σ_{min}^h discussed in the section 3.2.3, the threshold σ_{max}^h also matters. If $\sigma > \sigma_{max}^h$, increasing h reduces social welfare. In the economy with a high σ, the r_L^b is high in the initial equilibrium, which needs a small increase of h to achieve the target leverage. As a result, the welfare improvement is too little to compensate the loss induced by the sunk cost. Therefore, in this case, the macro-prudential policy makers choose the policy relating financial innovation when the policy target requires a high collateral requirement on CPI.

6 Conclusion

This paper analyzed the effect of belief disagreement on the welfare improvement induced by the policy tools which aim at mitigating default friction. The central feature of the model is that agents take risk of capital investment and borrow by selling collateralized loan contracts to lenders who hold heterogeneous beliefs. Particularly, lenders do not expect future capital returns as borrowers do, which means that the haircut is determined endogenously. Thus, I examine how this endogenous haircuts is influenced by the macro-prudential policies and the perturbations are through the collateral requirements as a loan contract term.

My results show that the tightness of borrowing constraint relating initial collateral requirements and belief distributions determines the effects of the policies on welfare. Also, if the financial innovation, collateral hedging, provides cheaper collateral, it mitigates collateral effects. This supports a Pareto-improving intervention which is an increase in the collateral requirement on CPI contracts combined with initial transfers. These results suggest that the identification of beliefs is crucial to macro-prudential interventions. However, if the set of observations is limited, the identification becomes problematic.[8] Considering the difficulties, the social planners should design the macro-prudential policies which apply to more environments captured by the belief distribution. Also, trades of CPI contracts provide more information about belief disagreement, though it is not sufficient for identifying each agent's belief.

My paper also implies that the insurance companies play an important role in implementing macro-prudential policies. Because they have credits to issue insurance contracts and economies of scale cuts the financial innovation cost, they act as moderates in the model. Even though governments which have the same advantages can partially substitute for private insurance companies, public sector frictions should be considered when designing optimal macro-economic policies (Williamson, 1986). Moreover, the competition of private intermediaries motivates them acquire more information of corporations before selling CPI contracts. As a result, the quality of capital investment is enhanced.

The analysis in this paper has not discussed whether CPI contracts can substitute other financial contracts, such as credit default swaps (CDSs), collateralized debt obligations (CDOs) and collateralized mortgage obligations. An extension of the model with richer contracts can analyze which types of financial contracts are endogenously traded in the equilibrium. I conjecture that trades of CPI contracts

[8]The problem will arise under uncertainty, when the market is incomplete and the payoffs of financial contracts are limited to a subspace of possible payoffs (Kübler & Polemarchakis, 2017).

will reduce the trades in some financial contract markets where the volatility of contract prices may hurt the trades in the commodity market. Therefore, the substitution effect is an interesting future direction to investigate.

Appendix

A. General equilibrium in the L-economy

In L-economy, there is no CPI. Agents 1 borrow money by issuing loan contracts and invest in capital. Agents 2 and 3 cannot get access to capital goods and only buy the loan contracts issued by agents 1. A collateral general equilibrium is a collection of prices, commodity holdings and contract trades $((\bar{m^b}), (\bar{c_s}^i, \bar{k}, \bar{\theta_b}^i))$ such that solves maximum problems as follow and market clearing conditions.

Maximum problem

Type 1

$$\max_{c_s^1,k,\theta_b^1} ln(c_0^1) + \beta^1 \sum_{s\in S_T} \pi_s^1 ln(c_s^1)$$

$$\text{subject to, } (c_0^1 + \alpha k - e_0^1) + m^b\theta_b^1 = 0$$

$$c_H^1 = e_H^1 + r_H^k k + \theta_b^1 \times min\{1, \phi r_H^k\}$$

$$c_L^1 = e_L^1 + r_L^k k + \theta_b^1 \times min\{1, \phi r_L^k\}$$

$$-\theta_b^1\phi = k$$

The first constraint reflects that money spent on commodities beyond the endowments in state 0 should be financed out of sale of loan contracts. The second and third constraints reflect that money spent on commodities beyond the endowments in either state $s \in \{H, L\}$ should be financed out of net revenue from dividend from contracts bought or sold in date 0. The fourth constraint reflects that agents of type 1 hold at least as much of pledged capital goods as they are required to post as collateral.

Set up the Lagrangian function:

$L = ln(c_0^1) + \beta^1(\pi_L^1 ln(c_L^1) + \pi_H^1 ln(c_H^1)) - \lambda_0^1(c_0^1 - e_0^1 + \alpha k + m^b\theta_b^1) - \lambda_L^1(c_L^1 - e_L^1 - r_L^k k - \theta_b^1\phi r_L^k) - \lambda_H^1(c_H^1 - e_H^1 - r_H^k k - \theta_b^1) + \mu^1(k + \phi\theta_b^1)$.

where $\lambda_s^1$are the Lagrangian multipliers for the budget constraints, and μ^1 is the Lagrangian multiplier for the the collateral constraint on capital goods.

In the equilibrium, for each agents i, $\bar{\lambda_s}^i = \partial u^i(\bar{c_s}^i)/\partial c_s^i, s \in S$. Also, I assume the present value of agents is $\bar{\gamma_s}^i = \bar{\lambda_s}^i/\bar{\lambda_0}^i, s \in S_T$. $\bar{\lambda_s}^i$ and $\bar{\gamma_s}^i$ will be used in the following appendix.

The Euler equations are

$$\theta_b^1 : \bar{m^b} = \bar{\gamma_L}^1\bar{\phi} r_L^k + \bar{\gamma_H}^1 + \frac{\bar{\phi}\bar{\mu}^1}{\bar{\lambda_0}^1}, \tag{A.1}$$

$$k : \alpha = \bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}. \tag{A.2}$$

Type 2

$$\max_{c_s^2,\theta_b^2} ln(c_0^2) + \beta^2 \sum_{s\in S_T} \pi_s^2 ln(c_s^2)$$

$$\text{subject to, } (c_0^2 - e_0^2) + m^b\theta_b^2 = 0$$

$$c_H^2 = e_H^2 + \theta_b^2 \times min\{1, \phi r_H^k\}$$

$$c_L^2 = e_L^2 + \theta_b^2 \times min\{1, \phi r_L^k\}$$

Set up the Lagrangian function:
$L = ln(c_0^2) + \beta^2(\pi_L^2 ln(c_L^2) + \pi_H^2 ln(c_H^2)) - \lambda_0^2(c_0^2 - e_0^2 + m^b\theta_b^2) - \lambda_L^2(c_L^2 - e_L^2 - \theta_b^2\phi r_L^k) - \lambda_H^2(c_H^2 - e_H^2 - \theta_b^2)$.
Hence, the Euler equation is

$$\theta_b^2 : \bar{m^b} = \bar{\gamma_L}^2\bar{\phi} r_L^k + \bar{\gamma_H}^2. \tag{A.3}$$

Type 3

$$\max_{c_s^3,\theta_b^3} ln(c_0^3) + \beta^3 \sum_{s\in S_T} \pi_s^3 ln(c_s^3)$$

$$\text{subject to, } (c_0^3 - e_0^3) + m^b\theta_b^3 = 0$$

$$c_H^3 = e_H^3 + \theta_b^3 \times min\{1, \phi r_H^k\}$$

$$c_L^3 = e_L^3 + \theta_b^3 \times min\{1, \phi r_L^k\}$$

Set up the Lagrangian function:
$L = ln(c_0^3) + \beta^3(\pi_L^3 ln(c_L^3) + \pi_H^3 ln(c_H^3)) - \lambda_0^3(c_0^3 - e_0^3 + m^b\theta_b^3) - \lambda_L^3(c_L^3 - e_L^3 - \theta_b^3\phi r_L^k) - \lambda_H^3(c_H^3 - e_H^3 - \theta_b^3)$
Hence, the Euler equation is

$$\theta_b^3 : \bar{m^b} = \bar{\gamma_L}^3\bar{\phi} r_L^k + \bar{\gamma_H}^3. \tag{A.4}$$

The general equilibrium $((\bar{m^b}), (\bar{c_s}^i, \bar{k}, \bar{\theta_b}^i))$ should satisfy budget constraints, four Euler equations, and one market clearing condition$-w^1\bar{\theta_b}^1 = w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3$.

B. General equilibrium in the CPI-economy

In the CPI-economy, there are CPI contracts. Agents 1 borrow money by issuing loan contracts and invest in capital goods and buy CPI contracts issued by agents 2. Agents 2 and 3 cannot get access to capital goods. Agents 2 buy the loan contracts issued by agents 1 and sell CPI contracts. Agents 3 take long position on loan contracts only. A collateral equilibrium is a collection of prices, commodity holdings and contract trades $((\bar{m^b}, \bar{m^j}), (\bar{c_s}^i, \bar{k}, \bar{\theta_b}^i, \bar{\theta_j}^i))$ such that solves maximum problems as follow and market clearing conditions.

Maximum problem

Type 1

$$\max_{c_s^1,k,\theta_j^1,\theta_b^1} ln(c_0^1) + \beta^1 \sum_{s\in S_T} \pi_s^1 ln(c_s^1)$$

$$\text{subject to, } (c_0^1 + \alpha k - e_0^1) + m^b\theta_b^1 + m^j\theta_j^1 = 0$$

$$c_L^1 = e_L^1 + r_L^k k + \theta_b^1 \times min\{1, \phi r_L^k + hr_L^j\} + \theta_j^1 \times r_L^j$$

$$c_H^1 = e_H^1 + r_H^k k + \theta_b^1 \times min\{1, \phi r_H^k + hr_H^j\} + \theta_j^1 \times r_H^j$$

$$-\theta_b^1\phi = k$$

$$-\theta_b^1 h = \theta_j^1, \text{ where } h \in [0,1]$$

The first three constraints are budget constraints, and the last two constraints are collateral constraints.

Set up the Lagrangian function:

$L = ln(c_0^1) + \beta^1(\pi_L^1 ln(c_L^1) + \pi_H^1 ln(c_H^1)) - \lambda_0^1(c_0^1 - e_0^1 + \alpha k + m^b\theta_b^1 + m^j\theta_j^1) - \lambda_L^1(c_L^1 - e_L^1 - r_L^k k - \theta_b^1(\phi r_L^k + hr_L^j) - r_L^j\theta_j^1) - \lambda_H^1(c_H^1 - e_H^1 - r_H^k k - \theta_b^1 - r_H^j\theta_j^1) + \mu^1(k + \phi\theta_b^1) + \eta^1(\theta_b^1 h + \theta_j^1)$

where η^1 is the Lagrangian multipliers for the the collateral constraint on CPI contracts.

The Euler equations are:

$$\theta_b^1 : \bar{m^b} = \bar{\gamma_L}^1(\bar{\phi} r_L^k + \bar{h} r_L^j) + \bar{\gamma_H}^1 + \frac{\bar{\phi}\bar{\mu}^1 + \bar{h}\bar{\eta}^1}{\bar{\lambda_0}^1}, \tag{B.1}$$

$$\theta_j^1 : \bar{m^j} = \bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1}, \tag{B.2}$$

$$k : \alpha = \bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}. \tag{B.3}$$

Type 2

$$\max_{c_s^2,\theta_j^2,\theta_b^2} ln(c_0^2) + \beta^2 \sum_{s\in S_T} \pi_s^2 ln(c_s^2)$$

$$\text{subject to, } (c_0^2 - e_0^2) + m^j\theta_j^2 + m^b\theta_b^2 - \varepsilon\theta_j^2 = 0$$

$$c_L^2 = e_L^2 + \theta_b^2 \times min\{1, \phi r_L^k + hr_L^j\} + \theta_j^2 \times r_L^j$$

$$c_H^2 = e_H^2 + \theta_b^2 \times min\{1, \phi r_H^k + hr_H^j\} + \theta_j^2 \times r_H^j$$

Set up the Lagrangian function:

$L = ln(c_0^2) + \beta^2(\pi_L^2 ln(c_L^2) + \pi_H^2 ln(c_H^2)) - \lambda_0^2(c_0^2 - e_0^2 + m^b\theta_b^2 + m^j\theta_j^2 - \varepsilon\theta_j^2) - \lambda_L^2(c_L^2 - e_L^2 - \theta_b^2(\phi r_L^k + hr_L^j) - r_L^j\theta_j^2) - \lambda_H^2(c_H^2 - e_H^2 - \theta_b^2 - r_H^j\theta_j^2)$

The Euler equations are:

$$\theta_b^2 : \bar{m^b} = \bar{\gamma_L}^2(\bar{\phi} r_L^k + \bar{h} r_L^j) + \bar{\gamma_H}^2, \tag{B.4}$$

$$\theta_j^2 : \bar{m^j} = \bar{\gamma_L}^2 r_L^j + \varepsilon. \tag{B.5}$$

Type 3

$$\max_{c_s^3,\theta_b^3} ln(c_0^3) + \beta^3 \sum_{s\in S_T} \pi_s^3 ln(c_s^3)$$

$$\text{subject to, } (c_0^3 - e_0^3) + m^b\theta_b^3 = 0$$

$$c_L^3 = e_L^3 + \theta_b^3 \times min\{1, \phi r_L^k + hr_L^j\}$$

$$c_H^3 = e_H^3 + \theta_b^3 \times min\{1, \phi r_H^k + hr_H^j\}$$

Set up the Lagrangian function:

$L = ln(c_0^3) + \beta^3(\pi_L^3 ln(c_L^3) + \pi_H^3 ln(c_H^3)) - \lambda_0^3(c_0^3 - e_0^3 + m^b\theta_b^3) - \lambda_L^3(c_L^3 - e_L^3 - \theta_b^3(\phi r_L^k + hr_L^j)) - \lambda_H^3(c_H^3 - e_H^3 - \theta_b^3)$

Hence, the Euler equation is

$$\theta_b^3 : \bar{m^b} = \bar{\gamma_L}^3(\bar{\phi} r_L^k + \bar{h} r_L^j) + \bar{\gamma_H}^3. \tag{B.6}$$

The general equilibrium $((\bar{m^b}, \bar{m^j}), (\bar{c_s}^i, \bar{k}, \bar{\theta_b}^i, \bar{\theta_j}^i))$ should satisfy budget constraints, six Euler equations, two market clearing conditions, $-w^1\bar{\theta_j}^1 = w^2\bar{\theta_j}^2$ and $-w^1\bar{\theta_b}^1 = w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3$.

C. Proofs

I. Macro-prudential perturbations in the L-economy

In the L-economy, the perturbation $d\phi$ at the initial state, where $d\phi > 0$ induce marginal changes at date 0, $(dc_0^i, dk, d\theta_b^i)$,with $\sum_{i=1}^{3} w^i d\theta_b^i = 0$, and then adjustments of the subsequent equilibrium plans and prices (dc_s^i, dm^b) around the equilibrium $(\bar{c_s}^i, \bar{\theta_b}^i, \bar{m^b})$.

Then, I compute the marginal change of consumption distribution of each type of agents, relative to the stationary competitive equilibrium, following a marginal change of the policy parameter $d\phi$. Because of market clearing conditions, the effect on social welfare does not require compute dm^b. Then, I compute the marginal changes of utilities of each type of agents and social welfare.

The change in agents i's marginal utility is given by

$$\frac{du^i}{\bar{\lambda_0}^i} \mid_{\phi=\bar{\phi}} = dc_0^i + \bar{\gamma_L}^i dc_L^i + \bar{\gamma_H}^i dc_H^i, \tag{C.1}$$

where $\bar{\lambda_0}^i = \partial u^i(\bar{c_0}^i)/\partial c_0^i > 0$.

Type 1

The change of type 1 consumption at date 0 is

$$dc_0^1 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^1 dm^b - \bar{m^b} d\theta_b^1 - adk. \tag{C.2}$$

Then, substitute (A.1), (A.2) into (C.1), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$
\begin{aligned}
dc_0^1 \mid_{\phi=\bar{\phi}} = & -\bar{\theta_b}^1 dm^b - (\bar{\gamma_L}^1 \bar{\phi} r_L^k + \bar{\gamma_H}^1 + \frac{\bar{\phi}\bar{\mu}^1}{\bar{\lambda_0}^1}) d\theta_b^1 - \\
& (\bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}) dk.
\end{aligned}
$$

Since both collateral constraint bind, $k = -\phi\theta_b^1$. Hence, the marginal changes $dk = -\bar{\phi} d\theta_b^1 - \bar{\theta_b}^1 d\phi$, which yields,

$$
\begin{aligned}
dc_0^1 \mid_{\phi=\bar{\phi}} = & -\bar{\theta_b}^1 dm^b - (\bar{\gamma_L}^1 \bar{\phi} r_L^k + \bar{\gamma_H}^1 + \frac{\bar{\phi}\bar{\mu}^1}{\bar{\lambda_0}^1}) d\theta_b^1 - \\
& (\bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1})(-\bar{\phi} d\theta_b^1 - \bar{\theta_b}^1 d\phi)
\end{aligned}
$$

After simplifying, I obtain

$$
dc_0^1 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^1 dm^b + \bar{\phi}\bar{\gamma_H}^1 r_H^k d\theta_b^1 - \bar{\gamma_H}^1 d\theta_b^1 + (\bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}) \bar{\theta_b}^1 d\phi \tag{C.3}
$$

The change of type 1 consumption in the bust state is

$$
dc_L^1 \mid_{\phi=\bar{\phi}} = \bar{\phi} r_L^k d\theta_b^1 + \bar{\theta_b}^1 r_L^k d\phi + r_L^k dk.
$$

Since collateral constraints require $dk = -\bar{\phi} d\theta_b^1 - \bar{\theta_b}^1 d\phi$, I obtain

$$
\begin{aligned}
dc_L^1 \mid_{\phi=\bar{\phi}} = & \bar{\phi} r_L^k d\theta_b^1 + \bar{\theta_b}^1 r_L^k d\phi + r_L^k(-\bar{\phi} d\theta_b^1 - \bar{\theta_b}^1 d\phi) \\
= & 0.
\end{aligned} \tag{C.4}
$$

The change of type 1 consumption in the boom state is

$$
dc_H^1 \mid_{\phi=\bar{\phi}} = d\theta_b^1 + r_H^k dk.
$$

Since $dk = -\bar{\phi} d\theta_b^1 - \bar{\theta_b}^1 d\phi$, I obtain

$$
dc_H^1 \mid_{\phi=\bar{\phi}} = d\theta_b^1 + r_H^k(-\bar{\phi} d\theta_b^1 - \bar{\theta_b}^1 d\phi). \tag{C.5}
$$

Substitute equations (C.3), (C.4) and (C.5) into equation (C.1), and the marginal change of agent 1's utility is

$$
\begin{aligned}
\frac{du^1}{\bar{\lambda_0}^1} \mid_{\phi=\bar{\phi}} = & dc_0^1 + \bar{\gamma_L}^1 dc_L^1 + \bar{\gamma_H}^1 dc_H^1 \\
= & -\bar{\theta_b}^1 dm^b + \bar{\phi}\bar{\gamma_H}^1 r_H^k d\theta_b^1 - \bar{\gamma_H}^1 d\theta_b^1 + \\
& (\bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}) \bar{\theta_b}^1 d\phi + \bar{\gamma_H}^1 (d\theta_b^1 + r_H^k(-\bar{\phi} d\theta_b^1 - \bar{\theta_b}^1 d\phi)).
\end{aligned}
$$

After simplifying,

$$
\frac{du^1}{\bar{\lambda_0}^1} \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^1 dm^b + (\bar{\gamma_L}^1 r_L^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}) \bar{\theta_b}^1 d\phi. \tag{C.6}
$$

Type 2

The change of type 2 consumption at date 0 is

$$dc_0^2 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^2 dm^b - \bar{m^b} d\theta_b^2. \tag{C.7}$$

Then, substitute (A.3) into (C.7), I obtain

$$dc_0^2 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^2 dm^b - (\bar{\gamma_L}^2 \bar{\phi} r_L^k + \bar{\gamma_H}^2) d\theta_b^2. \tag{C.8}$$

The change of type 2 consumption in the bust state is

$$dc_L^2 \mid_{\phi=\bar{\phi}} = \bar{\phi} r_L^k d\theta_b^2 + \bar{\theta_b}^2 r_L^k d\phi. \tag{C.9}$$

The change of type 2 consumption in the boom state is,

$$dc_H^2 \mid_{\phi=\bar{\phi}} = d\theta_b^2. \tag{C.10}$$

Substitute (C.8), (C.9) and (C.10) into equation (C.1), and the marginal change of agent 2's utility is

$$\begin{aligned}\frac{du^2}{\bar{\lambda_0}^2} \mid_{\phi=\bar{\phi}} =& dc_0^2 + \bar{\gamma_L}^2 dc_L^2 + \bar{\gamma_H}^2 dc_H^2 \\ =& -\bar{\theta_b}^2 dm^b - (\bar{\gamma_L}^2 \bar{\phi} r_L^k + \bar{\gamma_H}^2) d\theta_b^2 + \\ & \bar{\gamma_L}^2 (\bar{\phi} r_L^k d\theta_b^2 + \bar{\theta_b}^2 r_L^k d\phi) + \bar{\gamma_H}^2 d\theta_b^2.\end{aligned}$$

After simplifying,

$$\frac{du^2}{\bar{\lambda_0}^2} \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^2 dm^b + \bar{\gamma_L}^2 r_L^k \bar{\theta_b}^2 d\phi. \tag{C.11}$$

Type 3

The change of type 3 consumption at date 0 is

$$dc_0^3 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^3 dm^b - \bar{m^b} d\theta_b^3. \tag{C.12}$$

Then, substitute (A.4) into (C.12), I obtain

$$dc_0^3 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^3 dm^b - (\bar{\gamma_L}^3 \bar{\phi} r_L^k + \bar{\gamma_H}^3) d\theta_b^3. \tag{C.13}$$

The change of type 3 consumption in the bust state is

$$dc_L^3 \mid_{\phi=\bar{\phi}} = \bar{\phi} r_L^k d\theta_b^3 + \bar{\theta_b}^3 r_L^k d\phi. \tag{C.14}$$

The change of type 3 consumption in the boom state is,

$$dc_H^3 \mid_{\phi=\bar{\phi}} = d\theta_b^3. \tag{C.15}$$

Substitute (C.13), (C.14) and (C.15) into equation (C.1), and the marginal change of agent 3's utility is

$$\begin{aligned}\frac{du^3}{\bar{\lambda_0}^3}\mid_{\phi=\bar{\phi}}=&dc_0^3+\bar{\gamma_L}^3 dc_L^3+\bar{\gamma_H}^3 dc_H^3\\ =&-\bar{\theta_b}^3 dm^b-(\bar{\gamma_L}^3\bar{\phi}r_L^k+\bar{\gamma_H}^3)d\theta_b^3+\\ &\bar{\gamma_L}^3(\bar{\phi}r_L^k d\theta_b^3+\bar{\theta_b}^3 r_L^k d\phi)+\bar{\gamma_H}^3 d\theta_b^3.\end{aligned}$$

After simplifying,

$$\frac{du^3}{\bar{\lambda_0}^3}\mid_{\phi=\bar{\phi}}=-\bar{\theta_b}^3 dm^b+\bar{\gamma_L}^3 r_L^k\bar{\theta_b}^3 d\phi. \tag{C.16}$$

Social welfare

I add up equations (C.6), (C.11) and (C.16), and then

$$\begin{aligned}\sum_{i=1}^3 w^i\frac{du^i}{\bar{\lambda_0}^i}\mid_{\phi=\bar{\phi}}=&(w^1\frac{du^1}{\bar{\lambda_0}^1}+w^2\frac{du^2}{\bar{\lambda_0}^2}+w^3\frac{du^3}{\bar{\lambda_0}^3})\mid_{\phi=\bar{\phi}}\\ =&w^1(-\bar{\theta_b}^1 dm^b+(\bar{\gamma_L}^1 r_L^k+\frac{\bar{\mu}^1}{\bar{\lambda_0}^1})\bar{\theta_b}^1 d\phi)+\\ &w^2(-\bar{\theta_b}^2 dm^b+\bar{\gamma_L}^2 r_L^k\bar{\theta_b}^2 d\phi)+w^3(-\bar{\theta_b}^3 dm^b+\bar{\gamma_L}^3 r_L^k\bar{\theta_b}^3 d\phi)\\ =&-(w^1\bar{\theta_b}^1+w^2\bar{\theta_b}^2+w^3\bar{\theta_b}^3)dm^b+\\ &w^1\bar{\theta_b}^1(\bar{\gamma_L}^1 r_L^k+\frac{\bar{\mu}^1}{\bar{\lambda_0}^1})d\phi+w^2\bar{\theta_b}^2\bar{\gamma_L}^2 r_L^k d\phi+w^3\bar{\theta_b}^3\bar{\gamma_L}^3 r_L^k d\phi.\end{aligned}$$

When the loan contract market is clear, $-w^1\bar{\theta_b}^1=w^2\bar{\theta_b}^2+w^3\bar{\theta_b}^3$. Then I obtain,

$$\begin{aligned}\sum_{i=1}^3 w^i\frac{du^i}{\bar{\lambda_0}^i}\mid_{\phi=\bar{\phi}}=&w^1\bar{\theta_b}^1(\bar{\gamma_L}^1 r_L^k+\frac{\bar{\mu}^1}{\bar{\lambda_0}^1})d\phi+w^2\bar{\theta_b}^2\bar{\gamma_L}^2 r_L^k d\phi+w^3\bar{\theta_b}^3\bar{\gamma_L}^3 r_L^k d\phi\\ =&-(w^2\bar{\theta_b}^2+w^3\bar{\theta_b}^3)(\bar{\gamma_L}^1 r_L^k+\frac{\bar{\mu}^1}{\bar{\lambda_0}^1})d\phi+w^2\bar{\theta_b}^2\bar{\gamma_L}^2 r_L^k d\phi+w^3\bar{\theta_b}^3\bar{\gamma_L}^3 r_L^k d\phi.\end{aligned}$$

Hence,

$$(\sum_{i=1}^3 w^i\frac{du^i}{\bar{\lambda_0}^i}\mid_{\phi=\bar{\phi}})/d\phi=(w^2\bar{\theta_b}^2(\bar{\gamma_L}^2-\bar{\gamma_L}^1)+w^3\bar{\theta_b}^3(\bar{\gamma_L}^3-\bar{\gamma_L}^1))r_L^k-(w^2\bar{\theta_b}^2+w^3\bar{\theta_b}^3)\frac{\bar{\mu}^1}{\bar{\lambda_0}^1}.(7)$$

II. Macro-prudential perturbations in the CPI-economy

In the CPI-economy, the perturbation $dh>0$ at the initial state where it induces marginal changes at date 0, $(dc_0^i, dk, d\theta_b^i, d\theta_j^i)$,with $\sum_{i=1}^3 w^i d\theta_b^i=0$, $\sum_{i=1}^3 w^i d\theta_j^i=0$, and then adjustments of the subsequent equilibrium plans and prices (dc_s^i, dm^b, dm^j) around the equilibrium $(\bar{c_s}^i, \bar{\theta_b}^i, \bar{\theta_j}^i, \bar{m^b}, \bar{m^j})$.

Then, I compute the marginal change of consumption distribution of each type of agents, relative to the stationary competitive equilibrium, following a marginal change of the policy parameter dh. Because of market clearing conditions, the effect on social welfare does not require compute dm^b and dm^j. Then, I compute the marginal changes of utilities of each type of agents and social welfare.

The change in agents i's marginal utility is given by

$$\frac{du^i}{\bar{\lambda_0}^i} \mid_{h=\bar{h}}= dc_0^i + \bar{\gamma_L}^i dc_L^i + \bar{\gamma_H}^i dc_H^i, \tag{C.17}$$

where $\bar{\lambda_0}^i = 1/\bar{c_0}^i > 0$.

Type 1

The change of type 1 consumption at date 0 is

$$dc_0^1 \mid_{h=\bar{h}}= -\bar{\theta_b}^1 dm^b - \bar{m^b} d\theta_b^1 - \bar{\theta_j}^1 dm^j - \bar{m^j} d\theta_j^1 - adk. \tag{C.18}$$

Then, substitute (B.1), (B.2) and (B.3) into (C.18), which are from first order conditions for an optimum at the stationary competitive equilibrium:

$$\begin{aligned} dc_0^1 \mid_{h=\bar{h}}= &- \bar{\theta_b}^1 dm^b - \bar{\theta_j}^1 dm^j - (\bar{\gamma_L}^1(\bar{\phi} r_L^k + \bar{h} r_L^j) + \bar{\gamma_H}^1 + \frac{\bar{\mu}^1 + \bar{\eta}^1 \bar{h}}{\bar{\lambda_0}^1}) d\theta_b^1 - \\ &(\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1}) d\theta_j^1 - (\bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}) dk. \end{aligned}$$

Since both collateral constraint bind, $k = -\phi\theta_b^1$, $\theta_j^1 = -h\theta_b^1$. Hence, the marginal changes $dk = -\bar{\phi} d\theta_b^1$ and $d\theta_j^1 = -(\bar{\theta_b}^1 dh + \bar{h} d\theta_b^1)$, which yields

$$\begin{aligned} dc_0^1 \mid_{h=\bar{h}}= &- \bar{\theta_b}^1 dm^b - \bar{\theta_j}^1 dm^j - (\bar{\gamma_L}^1(\bar{\phi} r_L^k + \bar{h} r_L^j) + \bar{\gamma_H}^1 + \frac{\bar{\mu}^1 + \bar{\eta}^1 \bar{h}}{\bar{\lambda}_0^1}) d\theta_b^1 + \\ &(\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1}) \bar{\theta_b}^1 dh + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1}) \bar{h} d\theta_b^1 + (\bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}) dk. \end{aligned}$$

After simplifying, I obtain

$$dc_0^1 \mid_{h=\bar{h}}= -\bar{\theta_b}^1 dm^b - \bar{\theta_j}^1 dm^j - \bar{\gamma_H}^1 d\theta_b^1 - \bar{\gamma_H}^1 r_H^k dk + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1}) \bar{\theta_b}^1 dh. \tag{C.19}$$

The change of type 1 consumption in the bust state is

$$dc_L^1 \mid_{h=\bar{h}}= (\bar{\phi} r_L^k + \bar{h} r_L^j) d\theta_b^1 + \bar{\theta_b}^1 dh + r_L^k dk + r_L^j d\theta_j^1.$$

Since collateral constraints require $dk = -\bar{\phi} d\theta_b^1$ and $d\theta_j^1 = -(\bar{\theta_b}^1 dh + \bar{h} d\theta_b^1)$, I obtain

$$\begin{aligned} dc_L^1 \mid_{h=\bar{h}}=&(\bar{\phi} r_L^k + \bar{h} r_L^j) d\theta_b^1 + r_L^j \bar{\theta_b}^1 dh - \bar{\phi} r_L^k d\theta_b^1 - r_L^j (\bar{\theta_b}^1 dh + \bar{h} d\theta_b^1) \\ =&0. \end{aligned} \tag{C.20}$$

The change of type 1 consumption in the boom state is

$$dc_H^1 \mid_{h=\bar{h}}= d\theta_b^1 + r_H^k dk + r_H^j d\theta_j^1.$$

Since $r_H^j = 0$, I obtain

$$dc_H^1 \mid_{h=\bar{h}}= d\theta_b^1 + r_H^k dk. \tag{C.21}$$

Substitute equations (C.19), (C.20) and (C.21) into equation (C.18), and the marginal change of agent 1's utility is

$$\frac{du^1}{\bar{\lambda_0}^1}\mid_{h=\bar{h}} = dc_0^1 + \bar{\gamma_L}^1 dc_L^1 + \bar{\gamma_H}^1 dc_H^1$$
$$= -\bar{\theta_b}^1 dm^b - \bar{\theta_j}^1 dm^j - \bar{\gamma_H}^1 d\theta_b^1 - \bar{\gamma_H}^1 r_H^k dk +$$
$$(\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1})\bar{\theta_b}^1 dh + \bar{\gamma_H}^1(d\theta_b^1 + r_H^k dk).$$

After simplifying,

$$\frac{du^1}{\bar{\lambda_0}^1}\mid_{h=\bar{h}} = -\bar{\theta_b}^1 dm^b - \bar{\theta_j}^1 dm^j + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1})\bar{\theta_b}^1 dh. \tag{C.22}$$

Type 2

The change of type 2 consumption at date 0 is

$$dc_0^2\mid_{h=\bar{h}} = -\bar{\theta_b}^2 dm^b - \bar{m^b} d\theta_b^2 - \bar{\theta_j}^2 dm^j - \bar{m^j} d\theta_j^2 + \varepsilon d\theta_j^2. \tag{C.23}$$

Then, substitute (B.4) and (B.5) into (C.23), I obtain

$$\begin{aligned} dc_0^2\mid_{h=\bar{h}} &= -\bar{\theta_b}^2 dm^b - \bar{\theta_j}^2 dm^j - (\bar{\gamma_L}^2(\bar{\phi} r_L^k + \bar{h} r_L^j) + \bar{\gamma_H}) d\theta_b^2 - (\bar{\gamma_L}^2 r_L^j + \varepsilon) d\theta_j^2 + \varepsilon d\theta_j^2 \\ &= -\bar{\theta_b}^2 dm^b - \bar{\theta_j}^2 dm^j - (\bar{\gamma_L}^2(\bar{\phi} r_L^k + \bar{h} r_L^j) + \bar{\gamma_H}^2) d\theta_b^2 - \bar{\gamma_L}^2 r_L^j d\theta_j^2 \end{aligned} \tag{C.24}$$

Since agents 2 transfer the financial innovation cost to agents 1 by selling CPI at a higher price, ε does not influence c_0^2 and hence u^2.

The change of type 2 consumption in the bust state is

$$dc_L^2\mid_{h=\bar{h}} = (\bar{\phi} r_L^k + \bar{h} r_L^j) d\theta_b^2 + \bar{\theta_b}^2 r_L^j dh + r_L^j d\theta_j^2. \tag{C.25}$$

The change of type 2 consumption in the boom state is,

$$dc_H^2\mid_{h=\bar{h}} = d\theta_b^2 + r_H^j d\theta_j^2.$$

Since $r_H^j = 0$, I obtain

$$dc_H^2\mid_{h=\bar{h}} = d\theta_b^2. \tag{C.26}$$

Substitute (C.24), (C.25) and (C.26) into equation (C.18), and the marginal change of agent 2's utility is

$$\frac{du^2}{\bar{\lambda_0}^2}\mid_{h=\bar{h}} = dc_0^2 + \bar{\gamma_L}^2 dc_L^2 + \bar{\gamma_H}^2 dc_H^2$$
$$= -\bar{\theta_b}^2 dm^b - \bar{\theta_j}^2 dm^j - (\bar{\gamma_L}^2(\bar{\phi} r_L^k + \bar{h} r_L^j) - \bar{\gamma_H}^2) d\theta_b^2 - (\bar{\gamma_L}^2 r_L^j + \varepsilon) d\theta_j^2 + \varepsilon d\theta_j^2 +$$
$$\bar{\gamma_L}^2((\bar{\phi} r_L^k + \bar{h} r_L^j) d\theta_b^2 + \bar{\theta_b}^2 r_L^j dh + r_L^j d\theta_j^2) + \bar{\gamma_H}^2 d\theta_b^2.$$

After simplifying,

$$\frac{du^2}{\bar{\lambda_0}^2}\mid_{h=\bar{h}} = -\bar{\theta_b}^2 dm^b - \bar{\theta_j}^2 dm^j + \bar{\gamma_L}^2 r_L^j \bar{\theta_b}^2 dh. \tag{C.27}$$

Type 3

The change of type 3 consumption at date 0 is

$$dc_0^3 \mid_{h=\bar{h}} = -(\bar{\theta_b}^3)dm^b - (\bar{m^b})d\theta_b^3. \tag{C.28}$$

Then, substitute (B.6) in (C.28), which is from Euler equations for an optimum at the stationary competitive equilibrium, I obtain

$$dc_0^3 \mid_{h=\bar{h}} = -(\bar{\theta_b}^3)dm^b - (\bar{\gamma_L}^3(\bar{\phi}r_L^k + \bar{h}r_L^j) + \bar{\gamma_H}^3)d\theta_b^3. \tag{C.29}$$

The change of type 3 consumption in the bust state is

$$dc_L^3 \mid_{h=\bar{h}} = (\bar{\phi}r_L^k + \bar{h}r_L^j)d\theta_b^3 + \bar{\theta_b}^3 r_L^j dh. \tag{C.30}$$

The change of type 3 consumption in the boom state is,

$$dc_H^3 \mid_{h=\bar{h}} = d\theta_b^3. \tag{C.31}$$

Substitute (C.29), (C.30) and (C.31) into equation (C.18), and the changes of agent 3's utility:

$$\begin{aligned}\frac{du^3}{\bar{\lambda_0}^3} \mid_{h=\bar{h}} =& -\bar{\theta_b}^3 dm^b - (\bar{\gamma_L}^3(\bar{\phi}r_L^k + \bar{h}r_L^j) + \bar{\gamma_H}^3)d\theta_b^3 + \\ & \bar{\gamma_L}^3((\bar{\phi}r_L^k + \bar{h}r_L^j)d\theta_b^3 + \bar{\theta_b}^3 r_L^j dh) + \bar{\gamma_H}^3 d\theta_b^3.\end{aligned}$$

After simplifying, I obtain

$$\frac{du^3}{\bar{\lambda_0}^3} \mid_{h=\bar{h}} = -\bar{\theta_b}^3 dm^b + \bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 dh. \tag{C.32}$$

Social welfare

I add up equations (C.22), (C.27) and (C.32), and then

$$\begin{aligned}\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} \mid_{h=\bar{h}} =& (w^1 \frac{du^1}{\bar{\lambda_0}^1} + w^2 \frac{du^2}{\bar{\lambda_0}^2} + w^3 \frac{du^3}{\bar{\lambda_0}^3}) \mid_{h=\bar{h}} \\ =& w^1(-\bar{\theta_b}^1 dm^b - \bar{\theta_j}^1 dm^j + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1})\bar{\theta_b}^1 dh) + \\ & w^2(-\bar{\theta_b}^2 dm^b - \bar{\theta_j}^2 dm^j + \bar{\gamma_L}^2 r_L^j \bar{\theta_b}^2 dh) + w^3(-\bar{\theta_b}^3 dm^b + \bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 dh) \\ =& -(w^1\bar{\theta_b}^1 + w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3)dm^b - (w^1\bar{\theta_j}^1 + w^2\bar{\theta_j}^2)dm^j + \\ & w^1(\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1})\bar{\theta_b}^1 dh + w^2\bar{\gamma_L}^2 r_L^j \bar{\theta_b}^2 dh + w^3\bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 dh.\end{aligned}$$

When the contract markets are clear, $-w^1\bar{\theta_b}^1 = w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3$, $-w^1\bar{\theta_j}^1 = w^2\bar{\theta_j}^2$. Since $\bar{m^j} = \bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1} = \bar{\gamma_L}^2 r_L^j + \varepsilon$ and I add $w^3\bar{\gamma_L}^2 r_L^j \bar{\theta_b}^3 dh$ and subtract it, then I obtain

$$\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} |_{h=\bar{h}} = ((\bar{\gamma_L}^2 r_L^j + \varepsilon) w^1 \bar{\theta_b}^1 + \bar{\gamma_L}^2 r_L^j w^2 \bar{\theta_b}^2 + \bar{\gamma_L}^2 r_L^j w^3 \bar{\theta_b}^3) dh +$$
$$(w^3 \bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 - w^3 \bar{\gamma_L}^2 r_L^j \bar{\theta_b}^3) dh$$
$$= [\bar{\gamma_L}^2 r_L^j (w^1 \bar{\theta_b}^1 + w^2 \bar{\theta_b}^2 + w^3 \bar{\theta_b}^3) + \varepsilon w^1 \bar{\theta_b}^1] dh +$$
$$w^3 r_L^j \bar{\theta_b}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) dh.$$

Hence,

$$(\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} |_{h=\bar{h}} / dh) = w^3 r_L^j \bar{\theta_b}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) + \varepsilon w^1 \bar{\theta_b}^1. \tag{8}$$

III. Pareto-improving intervention

To give the Pareto-improving intervention, I demonstrate that everyone at date 0 can be made better off by appropriate redistribution dt^i. In particular, conditioning that

$$\varepsilon < -w^3 \bar{\theta_b}^3 r_L^j (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) / w^1 \bar{\theta_b}^1, \tag{C.33}$$

we compute transfers such that

$$\frac{du^i}{\bar{\lambda_0}^i} |_{h=\bar{h}} = \delta > 0,$$

where $\delta = (r_L^j w^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) \bar{\theta_b}^3 + \varepsilon w^1 \bar{\theta_b}^1) dh$

With the variables (dh, dt^i), agents i's change in utility can be expressed as

$$\frac{du^1}{\bar{\lambda_0}^i} |_{h=\bar{h}} = dt^1 + (-\bar{\theta_b}^1 dm^b + \bar{\theta_j}^1 dm^j + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1}) \bar{\theta_b}^1 dh), 9$$
$$\frac{du^2}{\bar{\lambda_0}^2} |_{h=\bar{h}} = dt^2 + (-\bar{\theta_b}^2 dm^b - \bar{\theta_j}^2 dm^j + \bar{\gamma_L}^2 r_L^j \bar{\theta_b}^2 dh), 10$$
$$\frac{du^3}{\bar{\lambda_0}^3} |_{h=\bar{h}} = dt^3 + (-\bar{\theta_b}^3 dm^b + \bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 dh).11$$

Initial agents are better off, if

$$dt^1 = -(-\bar{\theta_b}^1 dm^b + \bar{\theta_j}^1 dm^j + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1}) \bar{\theta_b}^1 dh) + \delta,$$
$$dt^2 = -(-\bar{\theta_b}^2 dm^b - (\bar{\theta_j}^2) dm^j + \bar{\gamma_L}^2 r_L^j \bar{\theta_b}^2 dh) + \delta,$$
$$dt^3 = -(-\bar{\theta_b}^3 dm^b + \bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 dh) + \delta.$$

Then, add up the three equations above, and I obtain

$$
\begin{aligned}
\sum_{i=1}^{3} w^i dt^i =& w^1(-(-\bar{\theta_b}^1 dm^b + \bar{\theta_j}^1 dm^j + (\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1})\bar{\theta_b}^1 dh) + \delta) + \\
& w^2(-(-\bar{\theta_b}^2 dm^b - \bar{\theta_j}^2 dm^j + \bar{\gamma_L}^2 r_L^j \bar{\theta_b}^2 dh) + \delta) + \\
& w^3(-(-\bar{\theta_b}^3 dm^b + \bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 dh) + \delta) \\
=& (w^1\bar{\theta_b}^1 + w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3) dm^b - (w^1\bar{\theta_j}^1 - w^2\bar{\theta_j}^2) dm^j - \\
& w^1(\bar{\gamma_L}^1 r_L^j + \frac{\bar{\eta}^1}{\bar{\lambda_0}^1})\bar{\theta_b}^1 dh - w^2\bar{\gamma_L}^2 r_L^j \bar{\theta_b}^2 dh - w^3\bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 dh + \\
& w^1\delta + w^2\delta + w^3\delta \\
=& -((\bar{\gamma_L}^2 r_L^j + \varepsilon) w^1\bar{\theta_b}^1 + \bar{\gamma_L}^2 r_L^j w^2 \bar{\theta_b}^2 + \bar{\gamma_L}^2 r_L^j w^3 \bar{\theta_b}^3) dh - \\
& (w^3\bar{\gamma_L}^3 r_L^j \bar{\theta_b}^3 - w^3\bar{\gamma_L}^2 r_L^j \bar{\theta_b}^3) dh + \delta \\
=& [-\bar{\gamma_L}^2 r_L^j (w^1\bar{\theta_b}^1 + w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3) - \varepsilon w^1 \bar{\theta_b}^1] dh - \\
& w^3 r_L^j \bar{\theta_b}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) dh + \delta \\
=& - w^3 r_L^j \bar{\theta_b}^3 (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) - \varepsilon w^1 \bar{\theta_b}^1 dh + \delta \\
=& 0.
\end{aligned}
$$

Finally, transfers add up to zero, $\sum_{i=1}^{3} w^i dt^i = 0$. It proves that (dh, dt^i) leads to a Pareto improvement and an increase of h induces a positive change of social welfare.

Moreover, the condition (C.33) is identical to

$$
-w^1\bar{\theta_b}^1 \bar{h}\varepsilon < w^3 \bar{\theta_b}^3 \bar{h} r_L^j (\bar{\gamma_L}^3 - \bar{\gamma_L}^2)
$$

Since $\bar{\theta_j}^1 = -\bar{\theta_b}^1 \bar{h}$,

$$
w^1\bar{\theta_j}^1 \varepsilon < w^3 \bar{\theta_b}^3 \bar{h} r_L^j (\bar{\gamma_L}^3 - \bar{\gamma_L}^2) 12
$$

Therefore, the Pareto-improvement requires that the financial innovation cost is lower than the net benefits of the society. It equals the benefits of agents 3 minus the loss of agents 2 when h increases.

D. Collateral effects in the No-default case.

If in the bust state loan sellers can deliver the promised consumption goods, $\bar{\phi} \geq 1/r_L^k$. Thus, the Euler equations (A.1), (A.2), (A.3) and (A.4) are changed into,

$$
\theta_b^1 : \bar{m}^b = \bar{\gamma_L}^1 + \bar{\gamma_H}^1 + \frac{\bar{\phi}\bar{\mu}^1}{\bar{\lambda_0}^1}, \tag{D.1}
$$

$$
k : \alpha = \bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}. \tag{D.2}
$$

$$
\theta_b^2 : \bar{m}^b = \bar{\gamma_L}^2 + \bar{\gamma_H}^2. \tag{D.3}
$$

$$
\theta_b^3 : \bar{m}^b = \bar{\gamma_L}^3 + \bar{\gamma_H}^3. \tag{D.4}
$$

In the L-economy, the perturbation $d\phi$ at the initial state, where $d\phi$ induces marginal changes at date 0, $(dc^i_{1,0}, dk, d\theta^i_b)$,with $\sum_{i=1}^{3} w^i d\theta^i_b = 0$, and then adjustments of the subsequent equilibrium plans and prices $(dc^i_{1,s}, dc^i_{2,s}, dm^b)$ around the equilibrium $(\bar{c_s}^i, \bar{p_s}, \bar{\theta_b}^i, \bar{m^b})$.

Type 1

The change of type 1 consumption at date 0 is

$$dc^1_0 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^1 dm^b - m^b d\theta^1_b - \alpha(-\bar{\phi} d\theta^1_b - \bar{\theta_b}^1 d\phi) \tag{D.5}$$

Substitute (D.1) and (D.2) into (D.5)

$$\begin{aligned} dc^1_0 \mid_{\phi=\bar{\phi}} = & -\bar{\theta_b}^1 dm^b - (\bar{\gamma_L}^1 + \bar{\gamma_H}^1 + \frac{\bar{\phi}\bar{\mu}^1}{\bar{\lambda_0}^1}) d\theta^1_b - \\ & (\bar{\gamma_L}^1 r^k_L + \bar{\gamma_H}^1 r^k_H + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1})(-\bar{\phi} d\theta^1_b - \bar{\theta_b}^1 d\phi) \end{aligned}$$

After simplifying, I obtain

$$dc^1_0 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^1 dm^b + \bar{\phi}\bar{\gamma_H}^1 r^k_H d\theta^1_b - \bar{\gamma_H}^1 d\theta^1_b + (\bar{\gamma_L}^1 r^k_L + \bar{\gamma_H}^1 r^k_H + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1})\bar{\theta_b}^1 d\phi \tag{D.6}$$

The change of type 1 consumption in the bust state is

$$dc^1_L \mid_{\phi=\bar{\phi}} = r^k_L \bar{\theta_b}^1 d\phi. \tag{D.7}$$

The change of type 1 consumption in the boom state is

$$dc^1_H \mid_{\phi=\bar{\phi}} = d\theta^1_b + r^k_H(-\bar{\phi} d\theta^1_b - \bar{\theta_b}^1 d\phi). \tag{D.8}$$

The marginal change of agent 1's utility is

$$\frac{du^1}{\bar{\lambda_0}^1} \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^1 dm^b + \bar{\theta_b}^1 \frac{\bar{\mu}^1}{\bar{\lambda_0}^1} d\phi \tag{D.9}$$

Type 2

The change of type 2 consumption at date 0 is

$$dc^2_0 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^2 dm^b - \bar{m}^b d\theta^2_b. \tag{D.10}$$

Then, substitute (D.3) into (D.9), I obtain

$$dc^2_0 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^2 dm^b - (\bar{\gamma_L}^2 + \bar{\gamma_H}^2) d\theta^2_b. \tag{D.11}$$

The change of type 2 consumption in the bust state is

$$dc^2_L \mid_{\phi=\bar{\phi}} = d\theta^2_b. \tag{D.12}$$

The change of type 2 consumption in the boom state is,

$$dc_H^2 \mid_{\phi=\bar{\phi}} = d\theta_b^2. \tag{D.13}$$

The marginal change of agent 2's utility is

$$\begin{aligned}\frac{du^2}{\bar{\lambda_0}^2} \mid_{\phi=\bar{\phi}} =& dc_0^2 + \bar{\gamma_L}^2 dc_L^2 + \bar{\gamma_H}^2 dc_H^2 \\ =& -\bar{\theta_b}^2 dm^b - (\bar{\gamma_L}^2 + \bar{\gamma_H}^2) d\theta_b^2 + \\ & \bar{\gamma_L}^2 d\theta_b^2 + \bar{\gamma_H}^2 d\theta_b^2.\end{aligned}$$

After simplifying,

$$\frac{du^2}{\bar{\lambda_0}^2} \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^2 dm^b. \tag{D.14}$$

Type 3

The change of type 3 consumption at date 0 is

$$dc_0^3 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^3 dm^b - \bar{m}^b d\theta_b^3. \tag{D.15}$$

Then, substitute (D.4) into (D.14), I obtain

$$dc_0^3 \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^3 dm^b - (\bar{\gamma_L}^3 + \bar{\gamma_H}^3) d\theta_b^3. \tag{D.16}$$

The change of type 3 consumption in the bust state is

$$dc_L^3 \mid_{\phi=\bar{\phi}} = d\theta_b^3. \tag{D.17}$$

The change of type 3 consumption in the boom state is,

$$dc_H^3 \mid_{\phi=\bar{\phi}} = d\theta_b^3. \tag{D.18}$$

The marginal change of agent 3's utility is

$$\begin{aligned}\frac{du^3}{\bar{\lambda_0}^3} \mid_{\phi=\bar{\phi}} =& dc_0^3 + \bar{\gamma_L}^3 dc_L^3 + \bar{\gamma_H}^3 dc_H^3 \\ =& -\bar{\theta_b}^3 dm^b - (\bar{\gamma_L}^3 + \bar{\gamma_H}^3) d\theta_b^3 + \\ & \bar{\gamma_L}^3 d\theta_b^3 + \bar{\gamma_H}^3 d\theta_b^3.\end{aligned}$$

After simplifying,

$$\frac{du^3}{\bar{\lambda_0}^3} \mid_{\phi=\bar{\phi}} = -\bar{\theta_b}^3 dm^b. \tag{D.19}$$

Social welfare

I add up equations (D.9), (D.14) and (D.19), and then

$$\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} \mid_{\phi=\bar{\phi}} = (w^1 \frac{du^1}{\bar{\lambda_0}^1} + w^2 \frac{du^2}{\bar{\lambda_0}^2} + w^3 \frac{du^3}{\bar{\lambda_0}^3}) \mid_{\phi=\bar{\phi}}$$
$$= w^1(-\bar{\theta_b}^1 dm^b + \frac{\bar{\mu}^1}{\bar{\lambda_0}^1}\bar{\theta_b}^1 d\phi) +$$
$$w^2(-\bar{\theta_b}^2 dm^b) + w^3(-\bar{\theta_b}^3 dm^b)$$
$$= -(w^1\bar{\theta_b}^1 + w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3) dm^b +$$
$$w^1\bar{\theta_b}^1 \frac{\bar{\mu}^1}{\bar{\lambda_0}^1} d\phi.$$

When the loan contract market is clear, $-w^1\bar{\theta_b}^1 = w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3$. Then I obtain,

$$\sum_{i=1}^{3} w^i \frac{du^i}{\bar{\lambda_0}^i} \mid_{\phi=\bar{\phi}} = -(w^2\bar{\theta_b}^2 + w^3\bar{\theta_b}^3) \frac{\bar{\mu}^1}{\bar{\lambda_0}^1} d\phi \tag{D.20}$$

Since $\bar{\theta_b}^1 < 0$, the positive sign of equation (D.20) requires $d\phi < 0$. This means that in the no-default case, there is no heterogeneity effects. Also, decreasing ϕ can mitigate the collateral effects, without influencing lenders' return. Compared with the possible default case, although the collateral effects are induced by increasing ϕ, social welfare is increased because the utilities of lenders are improved.

E. General Equilibrium in the First-best case

In this section, I will compare the collateral equilibrium with First-best equilibrium to complete the discussion.

In the complete market, agents do not need to hold collateral for issuing financial contracts, so agents 1 can sell Arrow securities, $Q^A = (A_u, A_d)$, to finance the capital investment, again without collateral requirements. Agents 2 and 3 can choose to buy or sell Arrow securities freely. Suppose the Arrow U securities promise $(1, 0)$, and the Arrow D securities promise $(0, 1)$. The maximum problem of agents 1 is,

$$\max_{c_s^1, k, \theta_q^1} ln(c_0^1) + \beta^1 \sum_{s \in S_T} \pi_s^1 ln(c_s^1)$$

$$\text{subject to, } (c_0^1 + \alpha k - e_0^1) + \sum_{q \in Q_A} m^q \theta_q^1 = 0$$

$$c_s^1 = e_s^1 + r_s^k k + \sum_{q \in Q_A} \theta_q^1 r_s^q$$

The Euler equations are:

$$k : \alpha = \bar{\gamma_L}^1 r_L^k + \bar{\gamma_H}^1 r_H^k$$
$$A_u : \bar{m^{A_u}} = \bar{\gamma_H}^1,$$

$$A_d : \bar{m^{A_d}} = \bar{\gamma_L}^1.$$

The maximum problem of agents 2 and 3 is, $i \in \{2, 3\}$,

$$\max_{c_s^i, \theta_q^i} ln(c_0^i) + \beta^i \sum_{s \in S_T} \pi_s^i ln(c_s^i)$$

$$\text{subject to, } (c_0^i - e_0^i) + \sum_{q \in Q_A} m^q \theta_q^i = 0$$

$$c_s^i = e_s^i + \sum_{q \in Q_A} \theta_q^i r_s^q$$

The Euler equations are:

$$A_u : \bar{m^{A_u}} = \bar{\gamma_H}^i,$$

$$A_d : \bar{m^{A_d}} = \bar{\gamma_L}^i.$$

The general equilibrium $((\bar{m^q}), (\bar{c_s}^i, \bar{k}, \bar{\theta_q}^i)), q \in Q^A$ should satisfy budget constraints, Euler equations, market clearing conditions, $\sum_{i=1}^{3} w^i \bar{\theta_q}^i = 0, \forall q \in Q^A$.

Then, in order to find the effects of the missing market, I compare the general equilibrium in the First-best case with a benchmark case where $\phi = 1.15$. In addition, I show another two cases where the specific collateral requirements make the investment equal to the First-best level. I use the fundamental values in Table 3. Also, the belief disagreement is 0.15 and the financial innovation cost in the CPI-economy is 0.001.

Table E.1: Comparison with the First-best case

	Benchmark	Changing ϕ	Changing h	First-best
ϕ	1.15	1.1322	1.15	-
h	-	-	0.5477	-
$Haircut$	0.4725	0.4706	0.4650	-
k	0.9490	0.9518	0.9518	0.9518
u^1	2.1837	2.1832	2.1842	2.1698
u^2	2.0122	2.0126	2.0120	2.0351
u^3	1.9591	1.9583	1.9589	1.9918
V	2.0513	2.0514	2.0516	2.0656

In the Table E.1, the last row shows that the welfare in the first-best case is the highest. Comparing the first and last columns, I find that the investment in the benchmark case is less than that in the First-best case.[9] The result of under-investment is different from the result of Fostel and Geanakoplos (2016). The reason is that I assume optimists can issue risky loan contracts, while they study the general equilibrium with riskless loans. In addition, the first and second columns show that in the L-economy, decreasing ϕ increases the investment to the First-best level. The $Haircut$ is lower and social welfare is improved. In contrast, the third column illustrates that the First-best investment is achieved by increasing h. Both the decrease of $Haircut$ and the welfare improvement are more, compared with that of decreasing ϕ. This proves that CPI contracts make the financial market more complete and drive the welfare closer to the First-best level.

F. $\mathbf{R_L^b}$ in the initial equilibrium

Figure F.1 and figure F.2 demonstrate that the policy target $R_L^b = 0$ in the simulation 2 and 3 requires to increase R_L^b in the initial equilibrium. Also, tightening collateral constraints leads R_L^b to go up.

[9] The incomplete market with collateral constraints do not always reduce investment. If ϕ is lower, the investment is more than the First-best case.

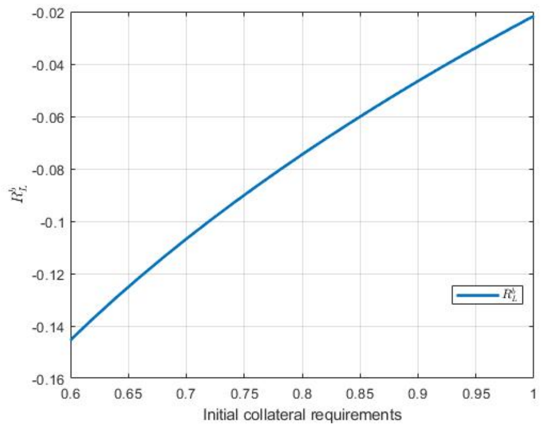


Figure 20: R_L^b for varying $\bar{\phi}$ in the initial equilibrium

Notes: $\sigma = 0.15$.

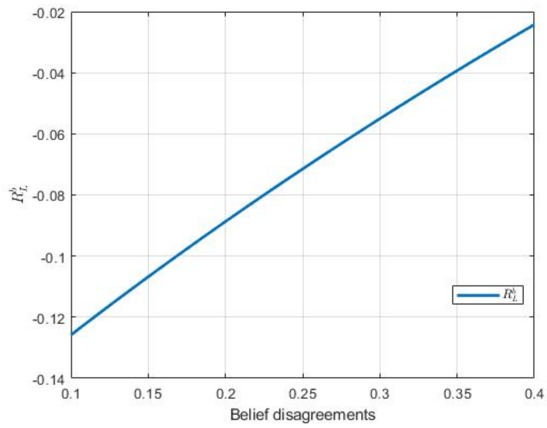


Figure 21: R_L^b for varying σ in the initial equilibrium

Notes: $\bar{\phi} = 0.7$.

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